Reinforcement Learning & Markov Decision Processes

Lucas Janson

CS/Stat 184(0): Introduction to Reinforcement Learning

Fall 2024

Today

- Logistics (Welcome!)
- Overview of RL
- Markov Decision Processes
    - Problem statement
    - Policy Evaluation

Course staff introductions

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- Homework 0 is posted!
  - This is “review” homework for material you should be familiar with to take the course.

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http://lucasjanson.fas.harvard.edu/CS_Stat_184_0.html

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- Project (30%): 2-3 people per project. Will be empirical.

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- Regrading: ask us in writing on Ed within a week

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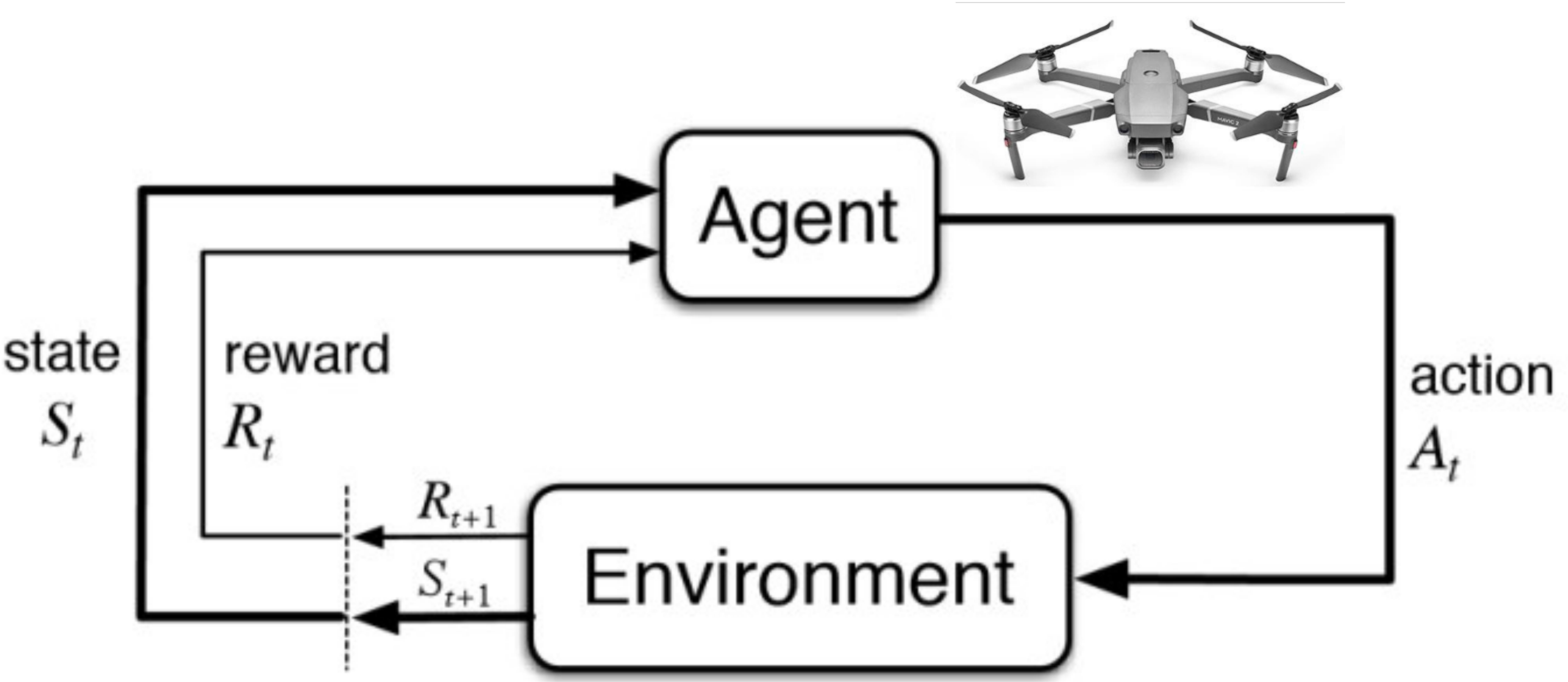
The RL Setting, basically

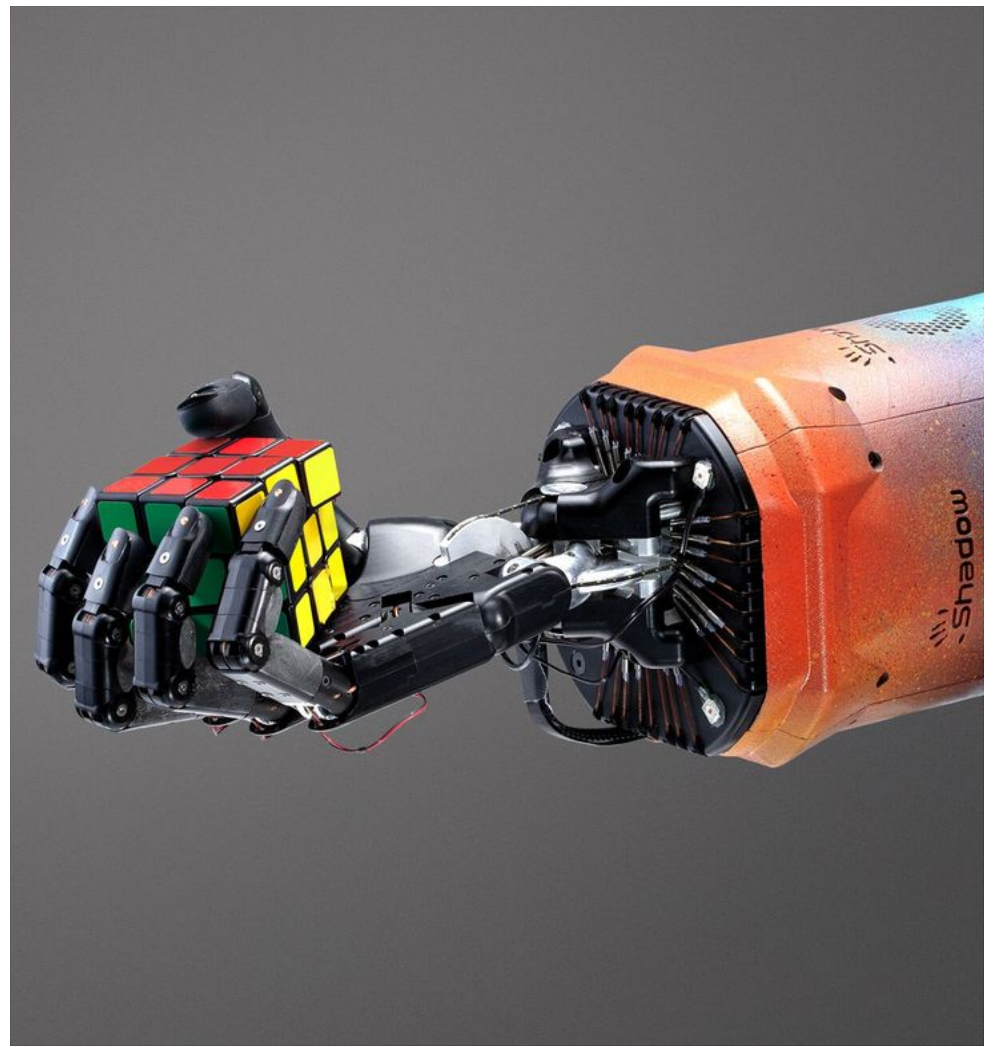


Many RL Successes

Online advertising

[AlphaZero, Silver et.al, 17]

[OpenAI Five, 18]



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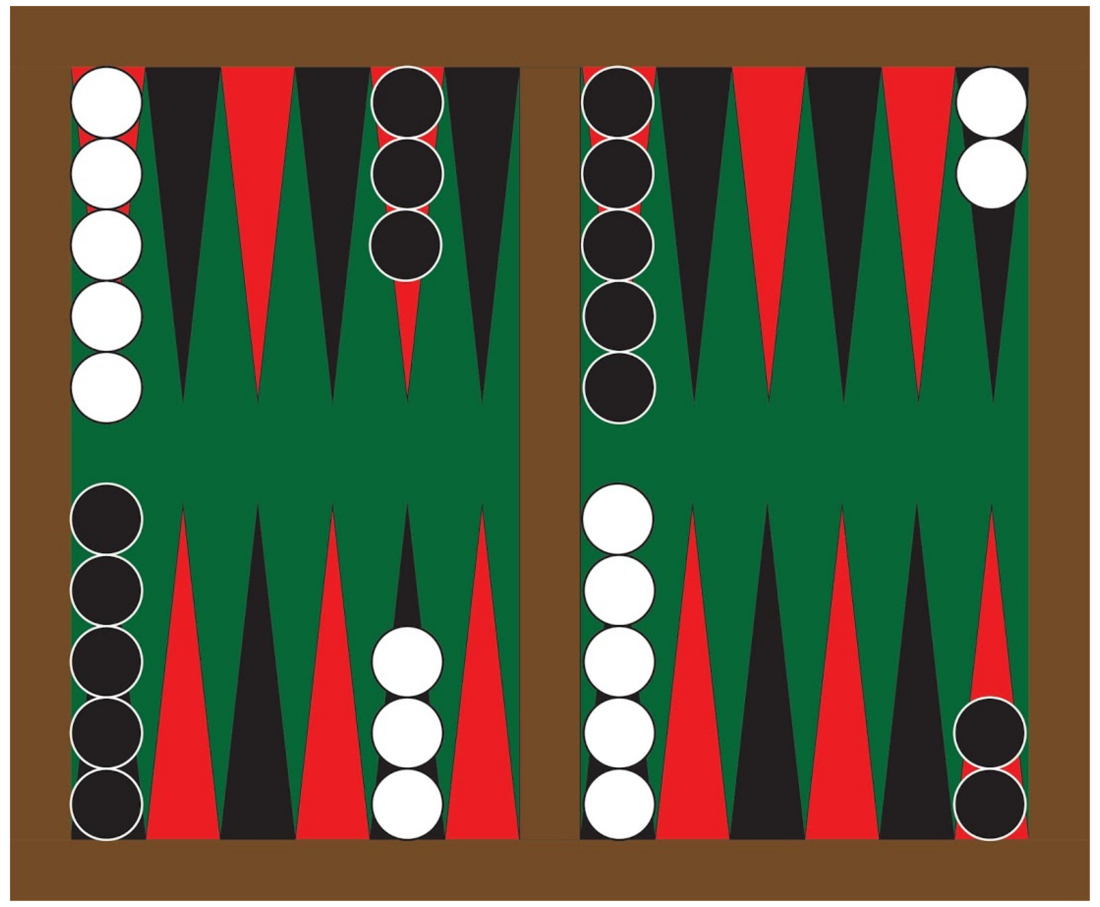

TD GAMMON [Tesauro 95]

Supply Chains [Madeka et al '23]

Many Future RL Challenges

Vs Other Settings

	Learn from Experience	Generalize	Interactive	Exploration	Credit assignment
Supervised Learning	✔	✔			
Bandits (“horizon 1”-RL)	✔	✔	✔	✔	
“Full” Reinforcement Learning	✔	✔	✔	✔	✔

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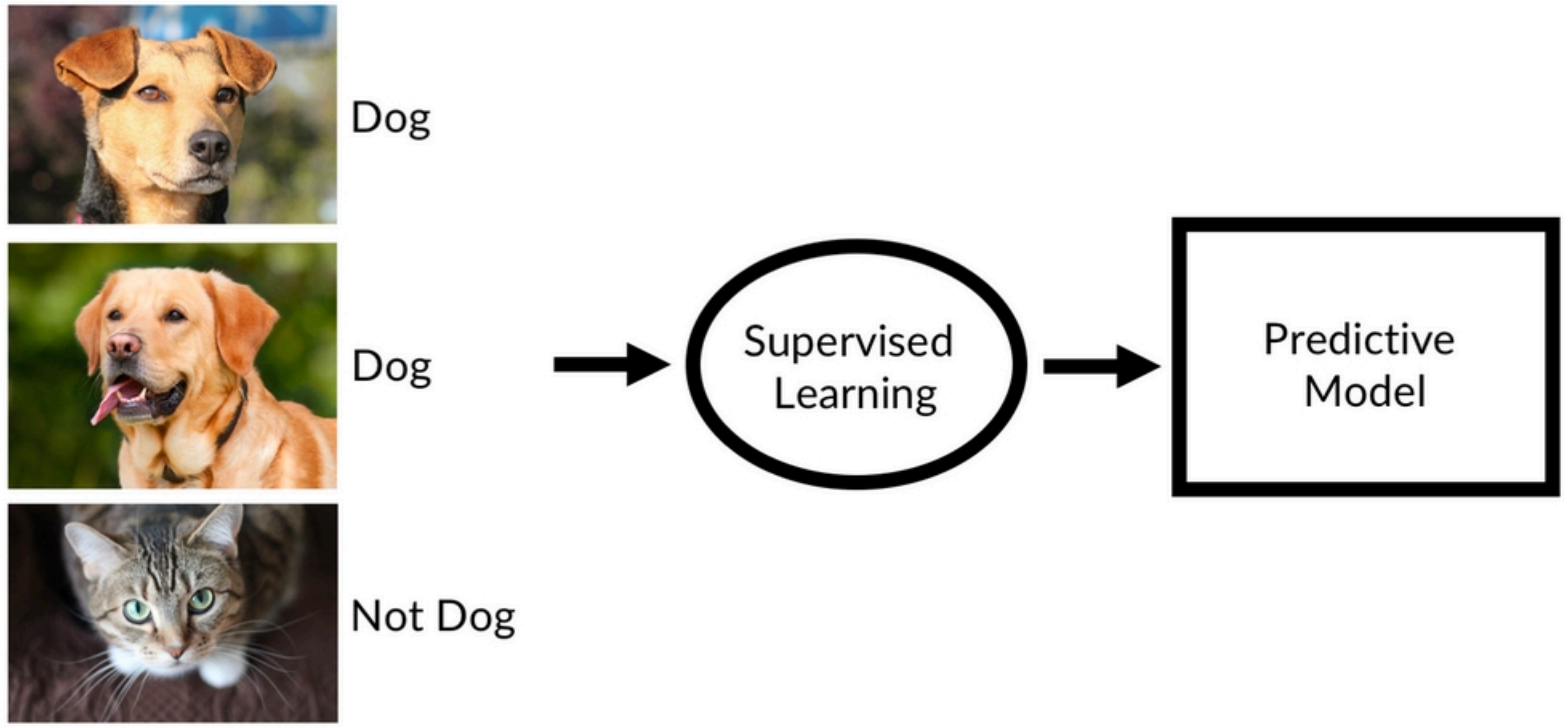

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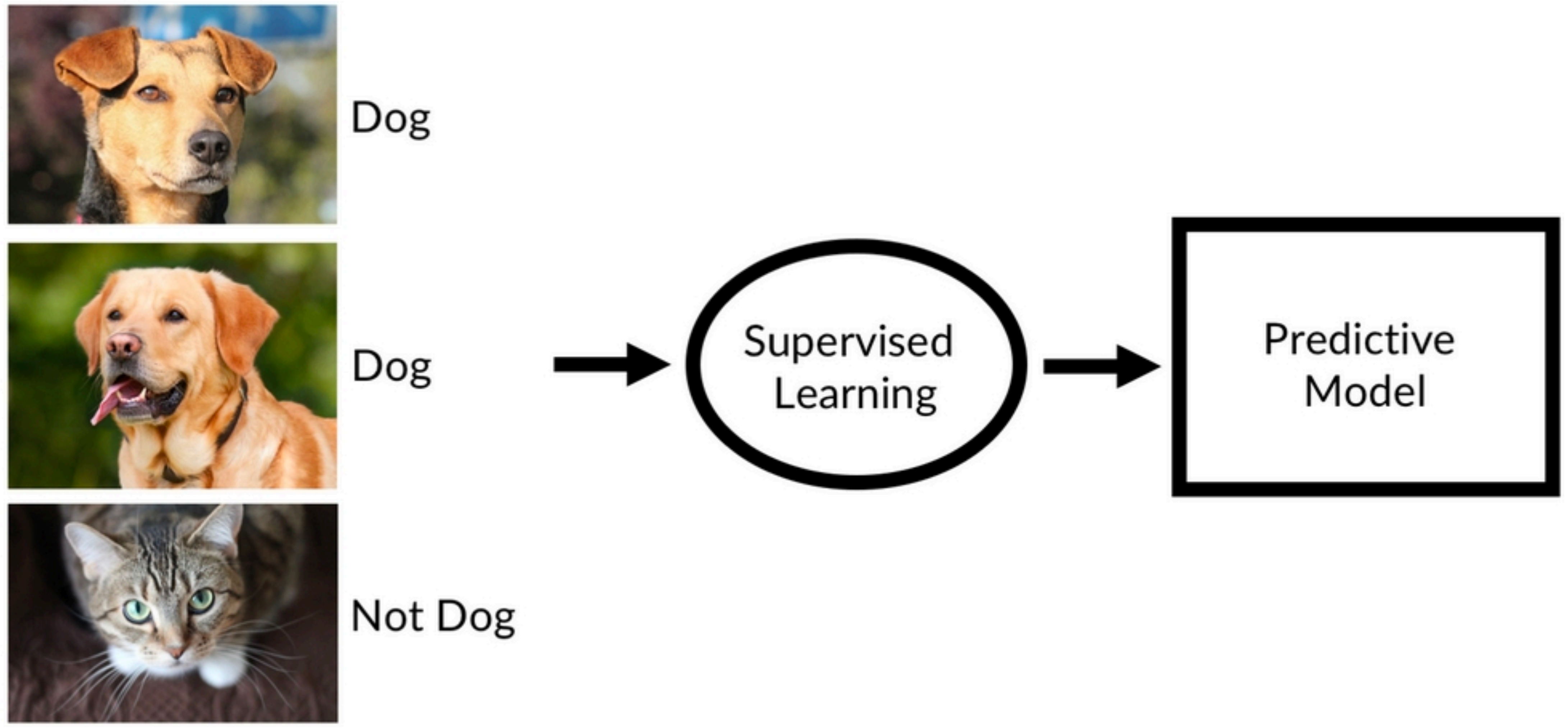

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- Surprising how much you can learn *without* any knowledge of supervised learning
    - In some sense, the fundamentals of RL are orthogonal to supervised learning

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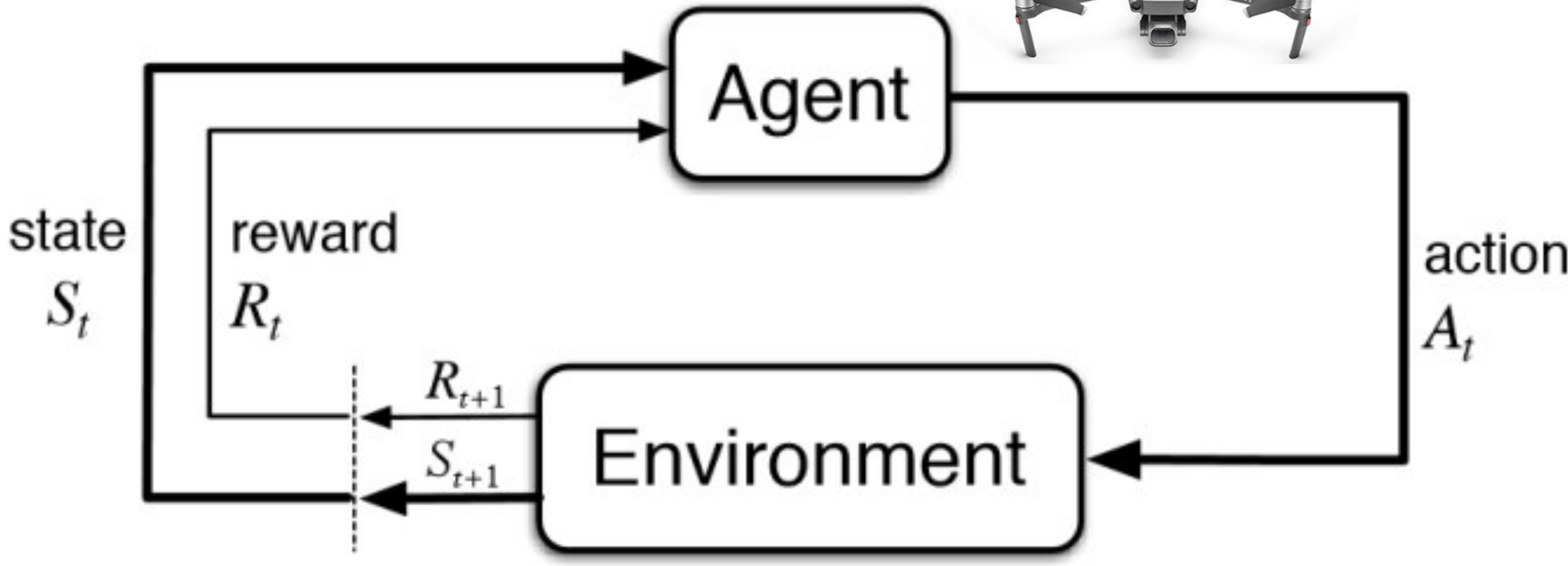

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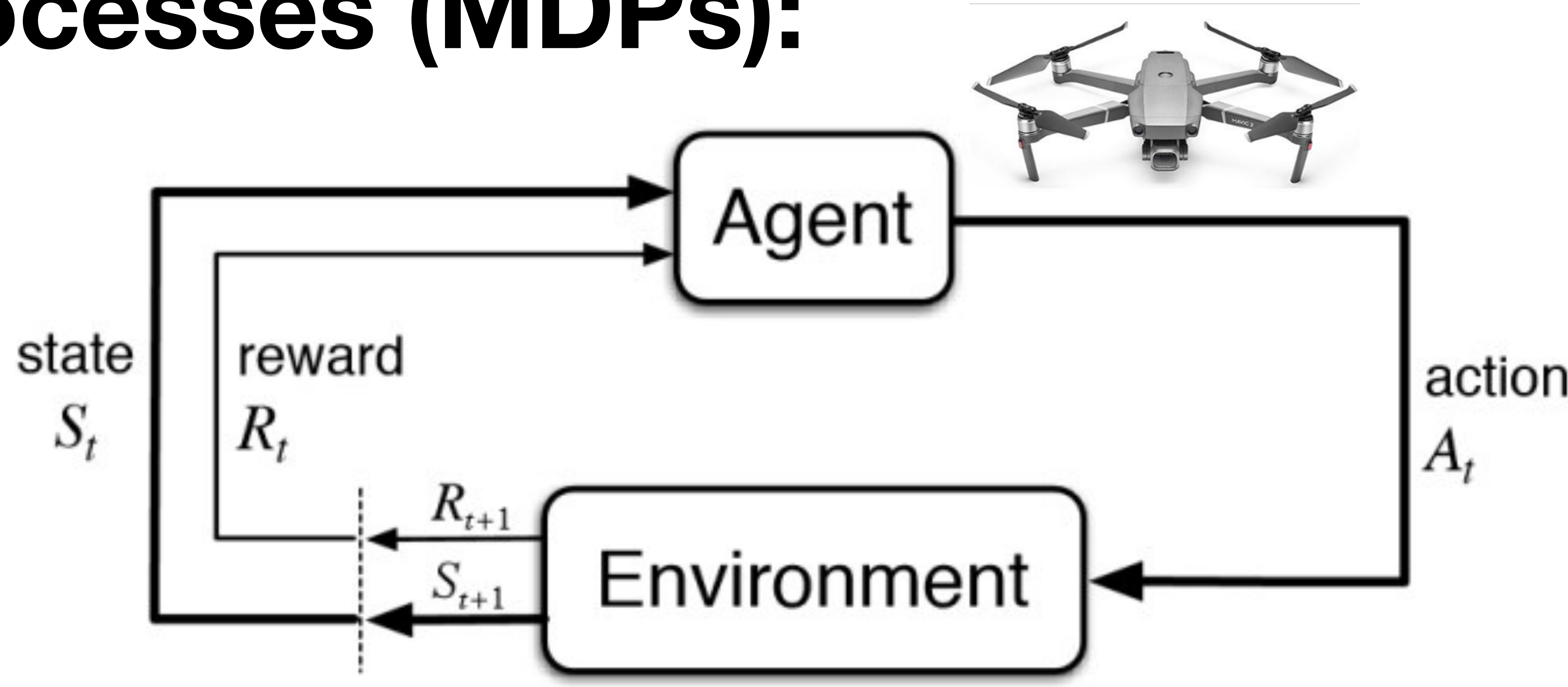

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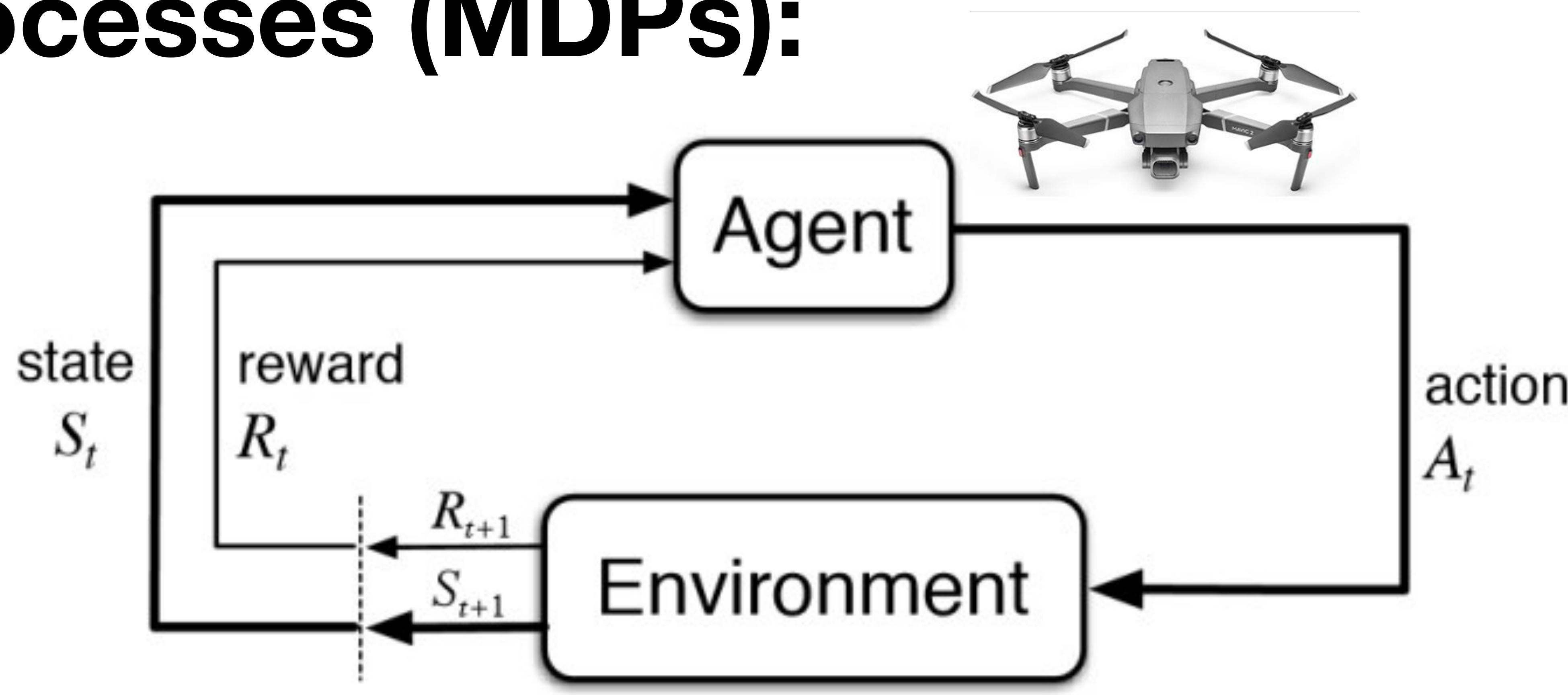

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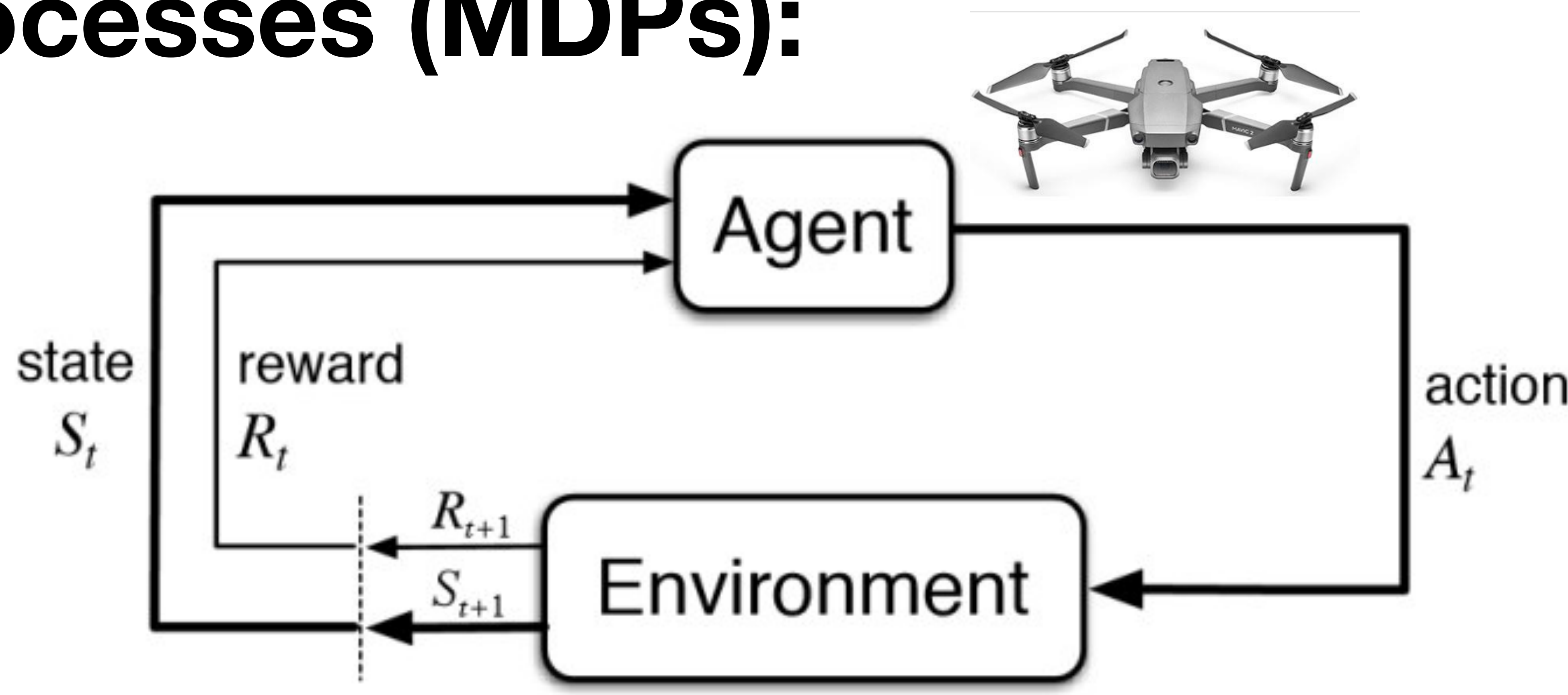

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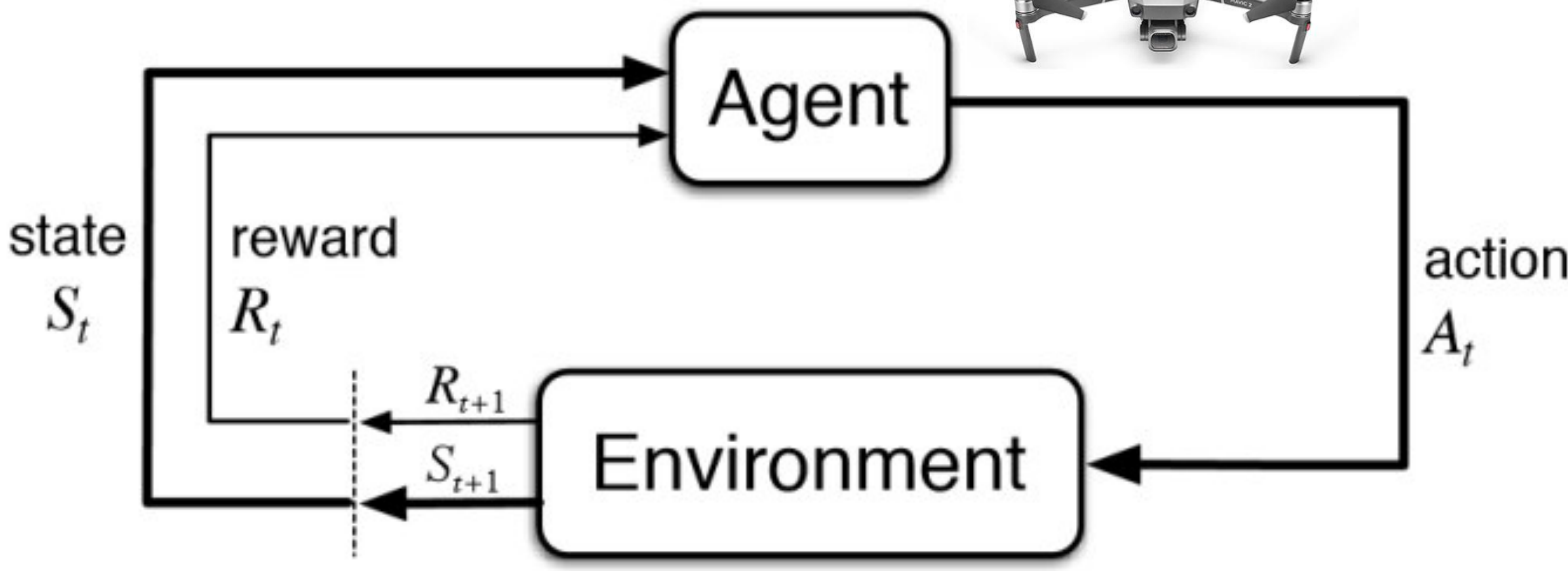

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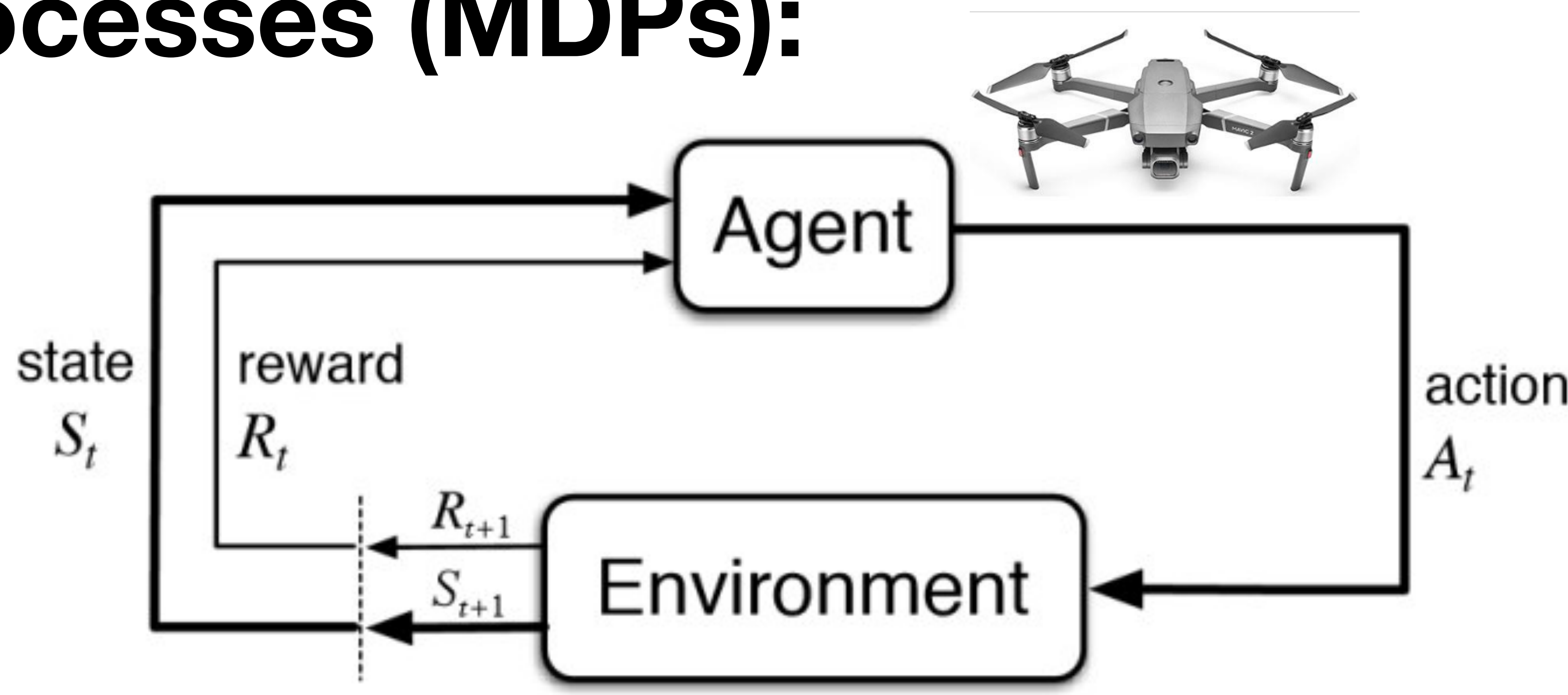

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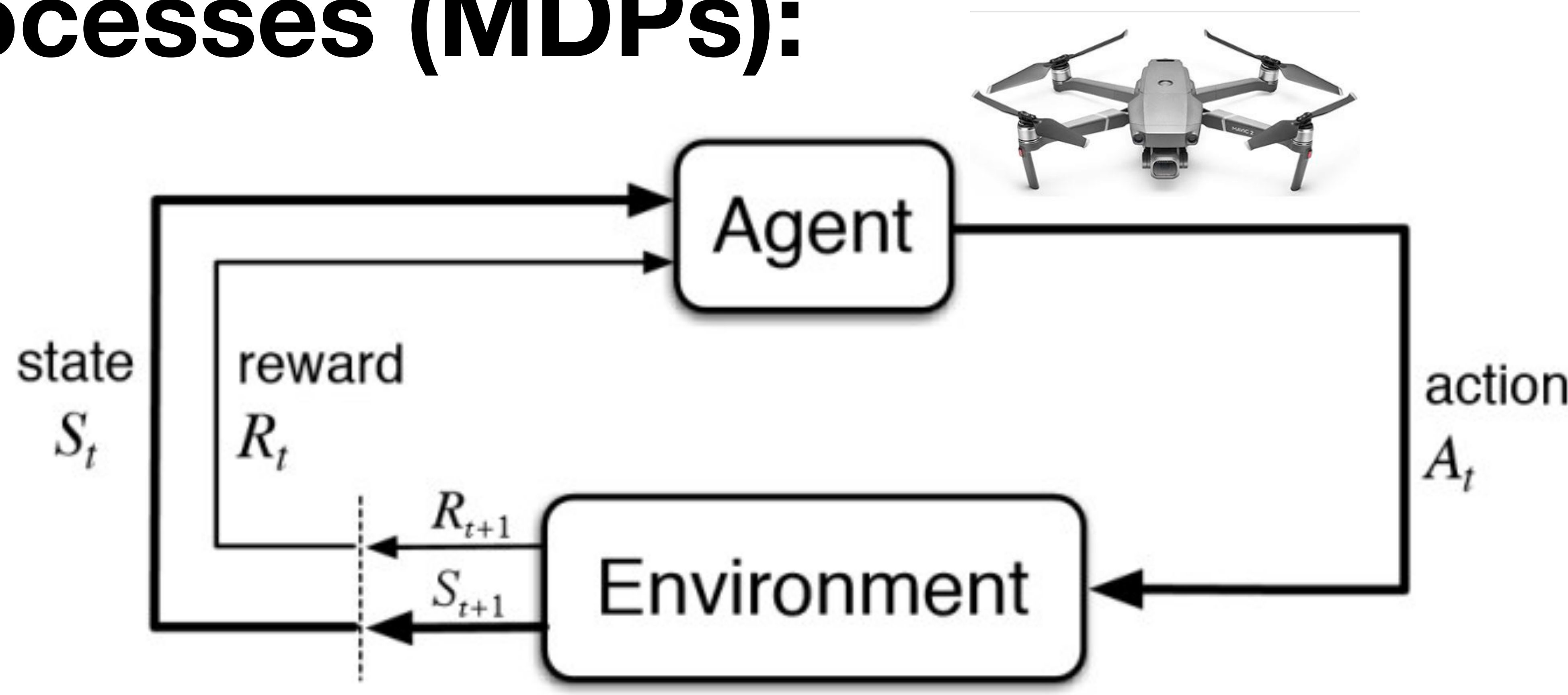

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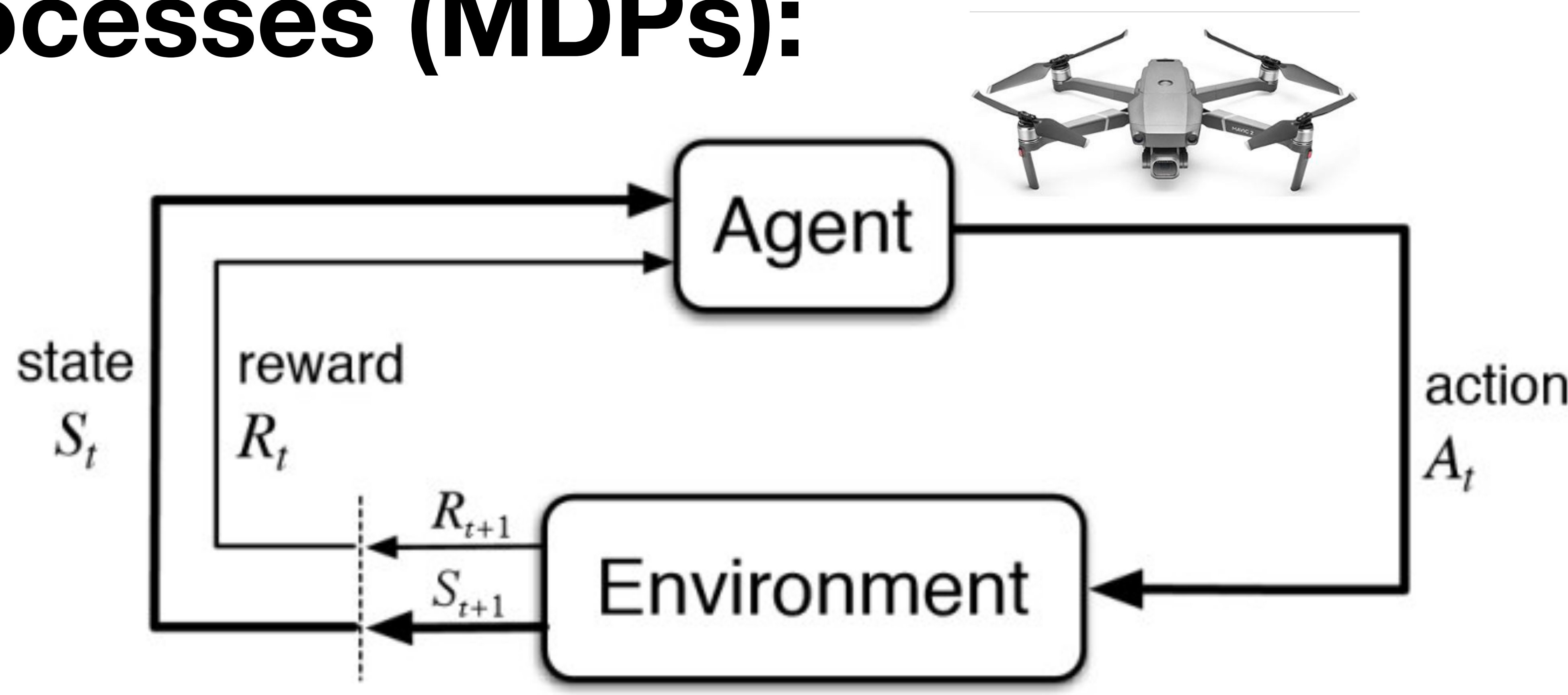

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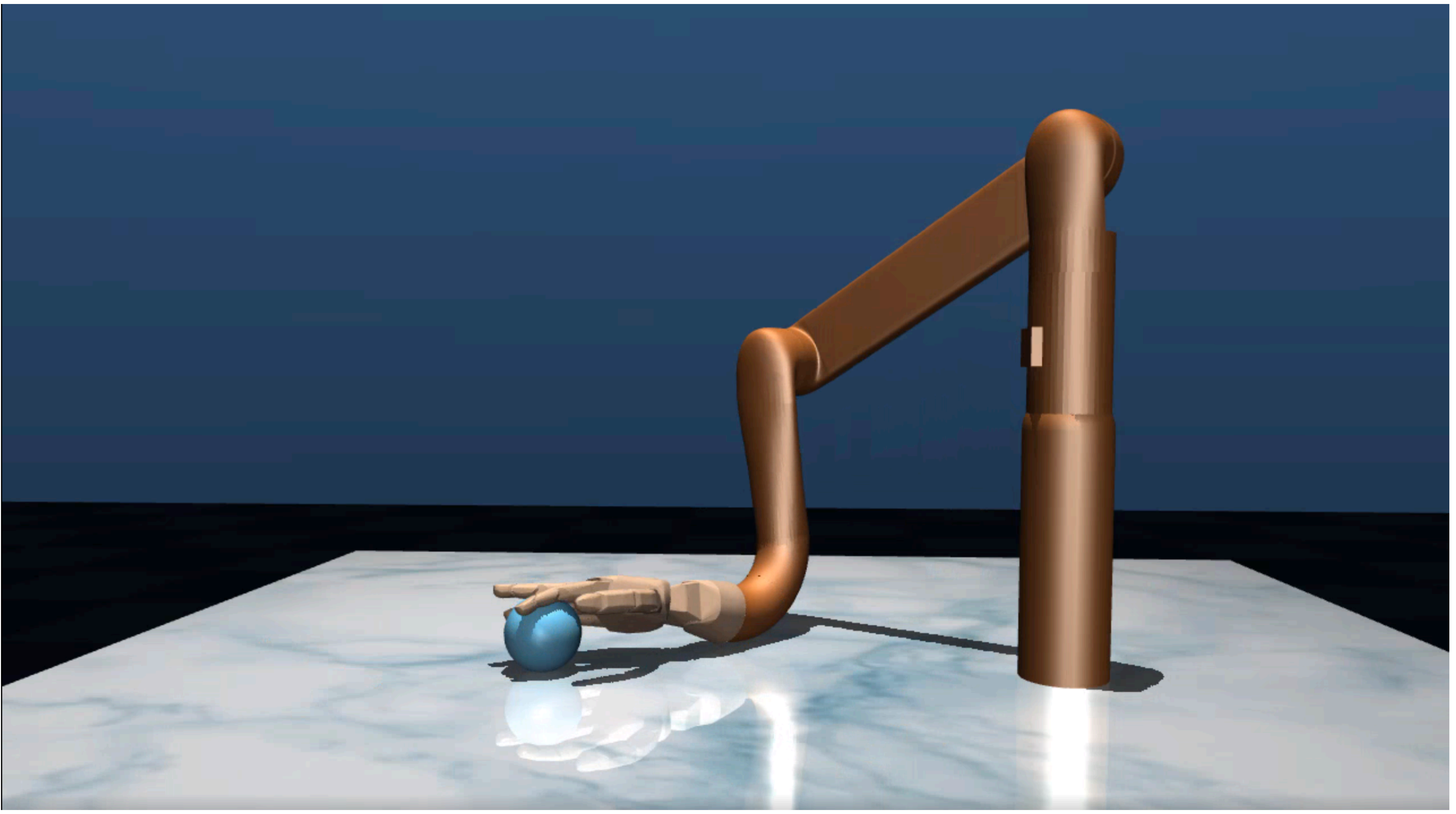
  - A time horizon $H \in \mathbb{N}$

Example:
robot hand needs to pick the ball and hold it in a goal (x,y,z) position



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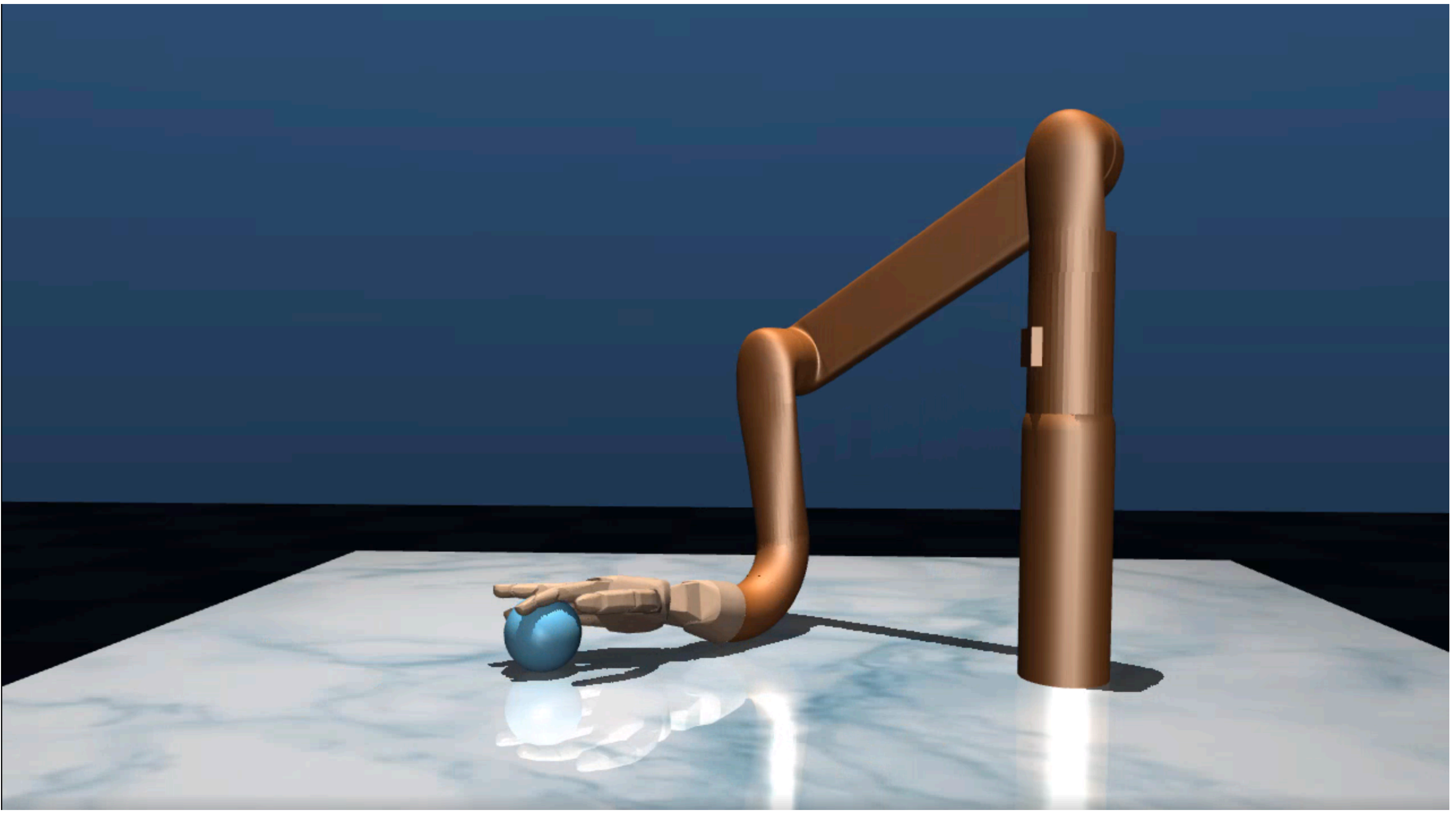

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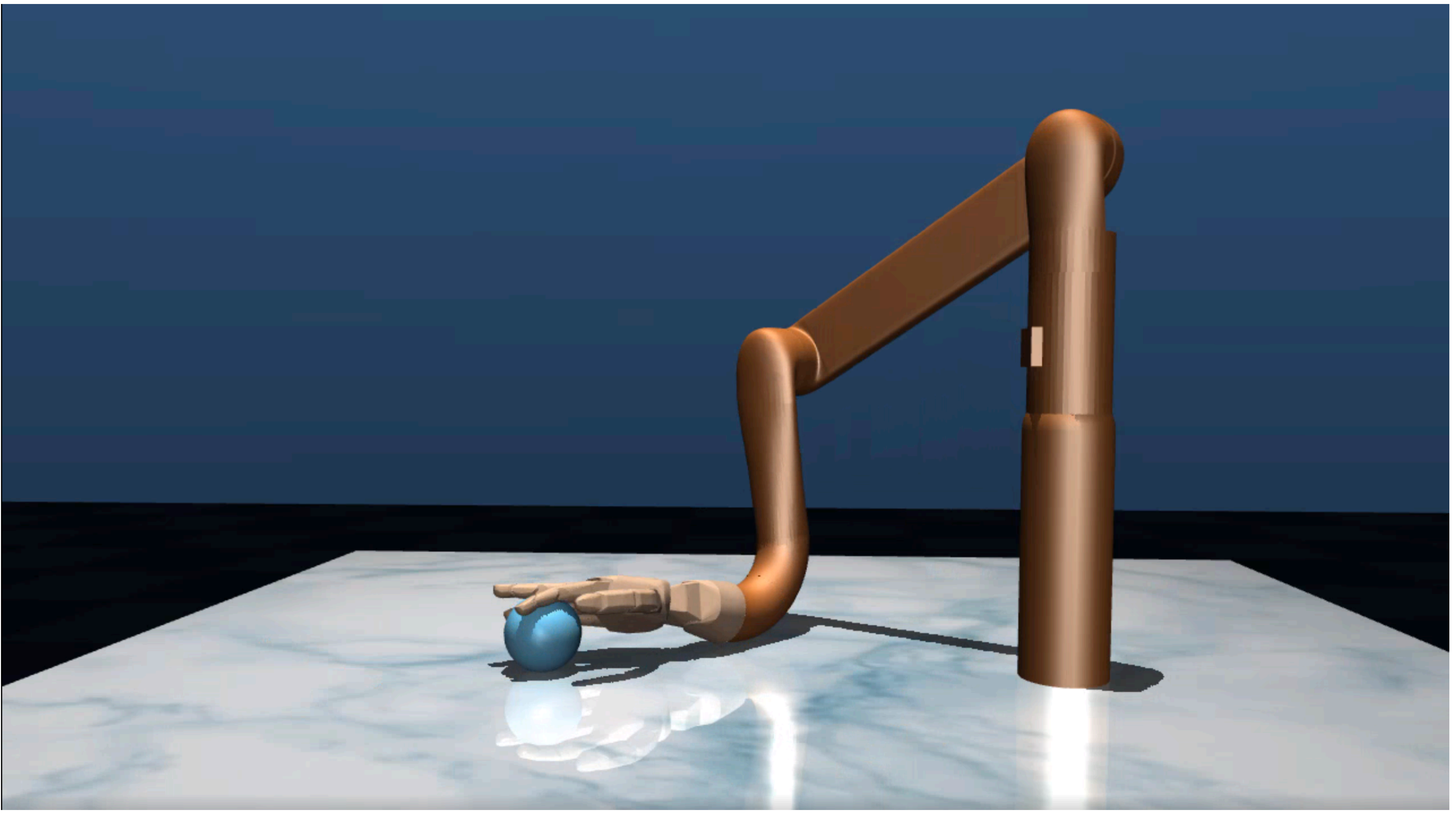

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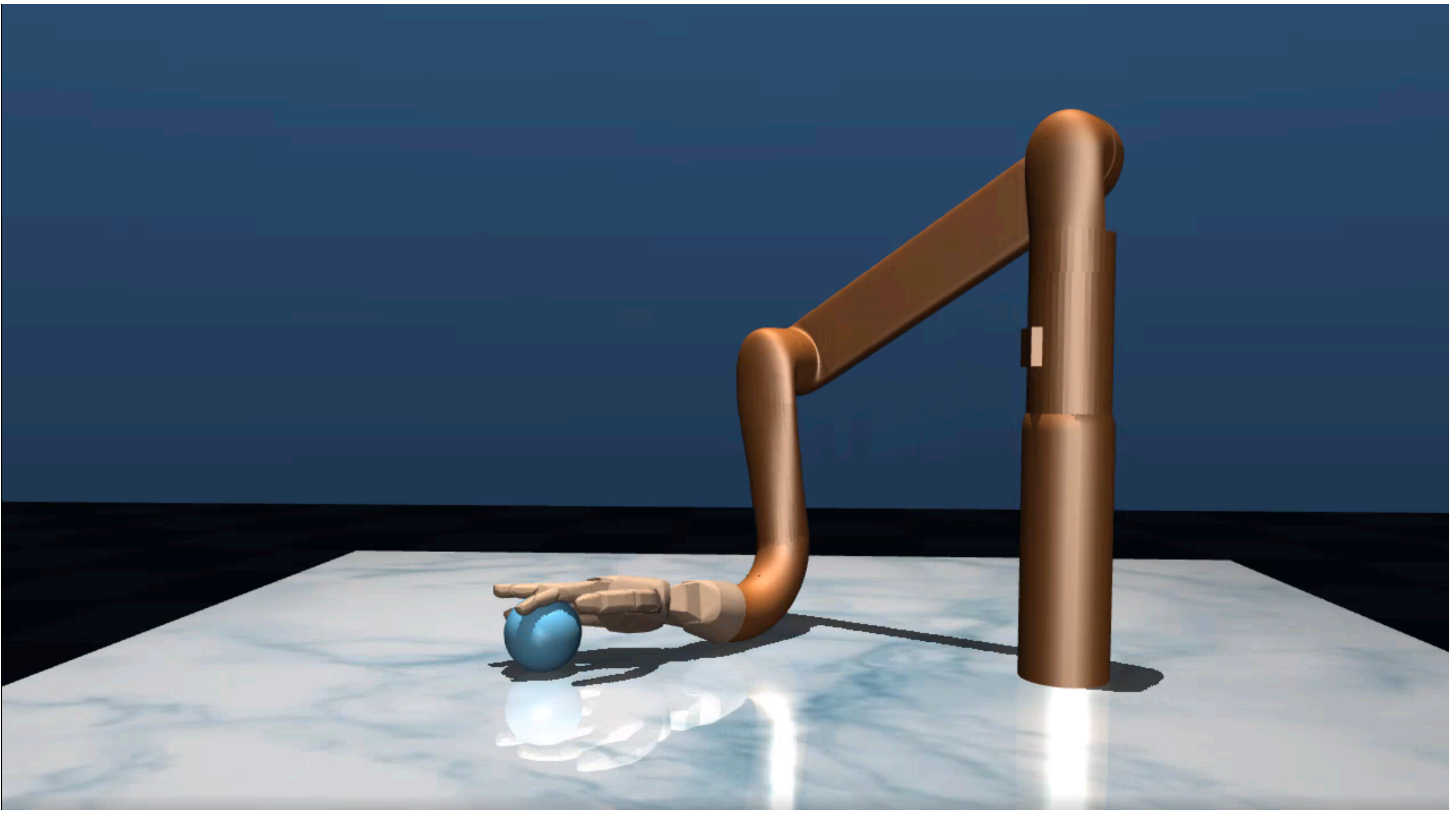

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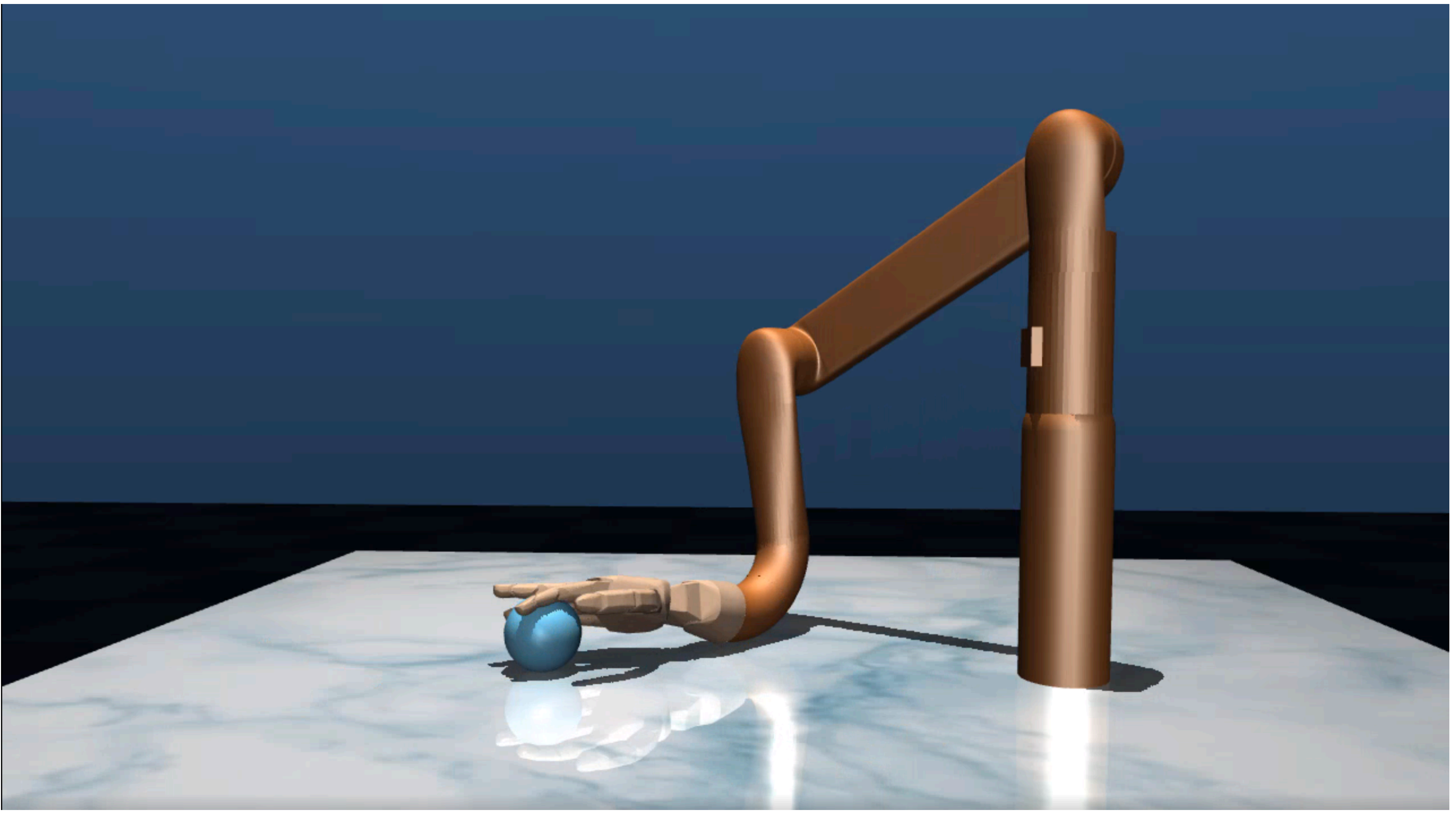

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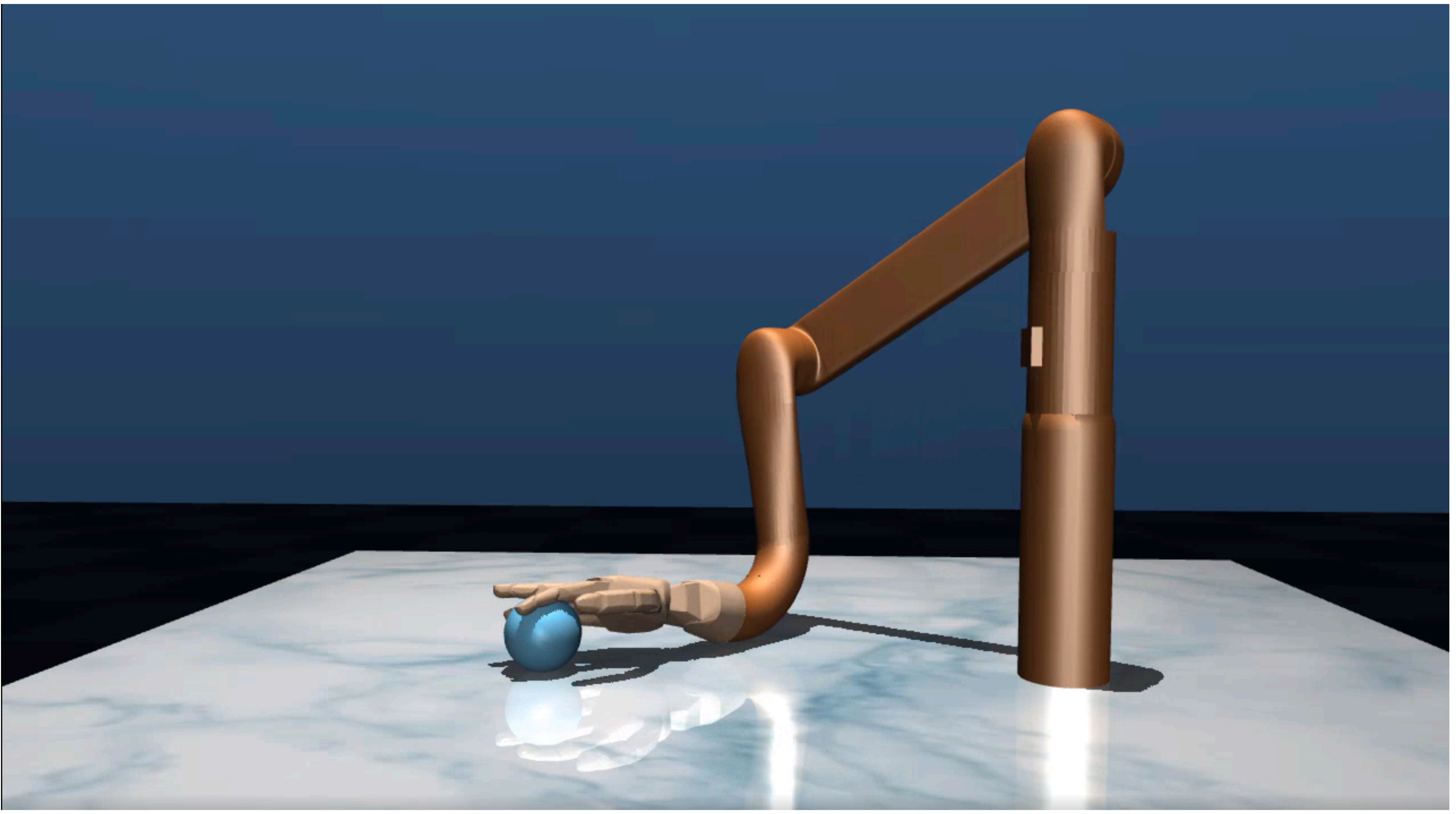

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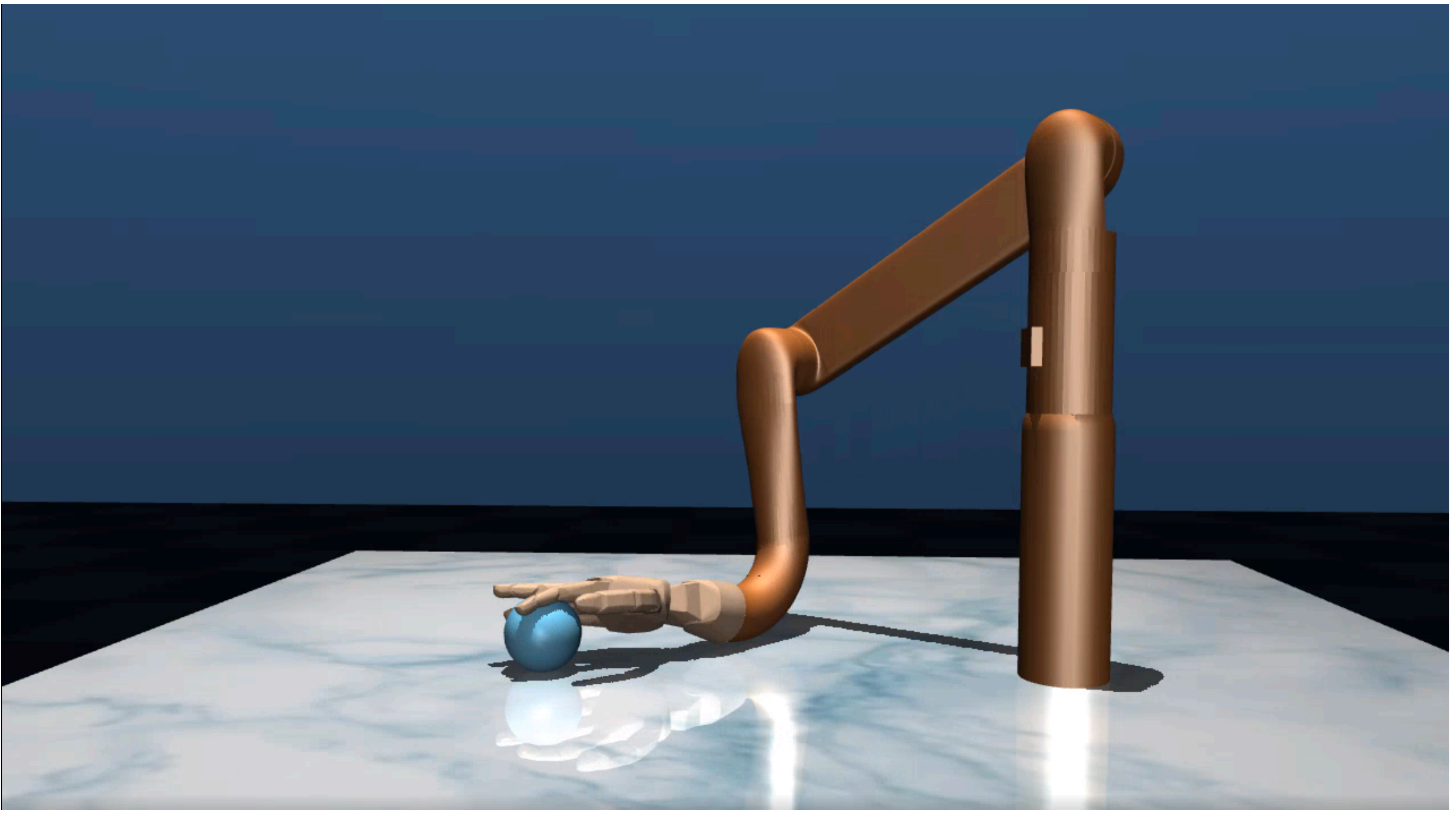

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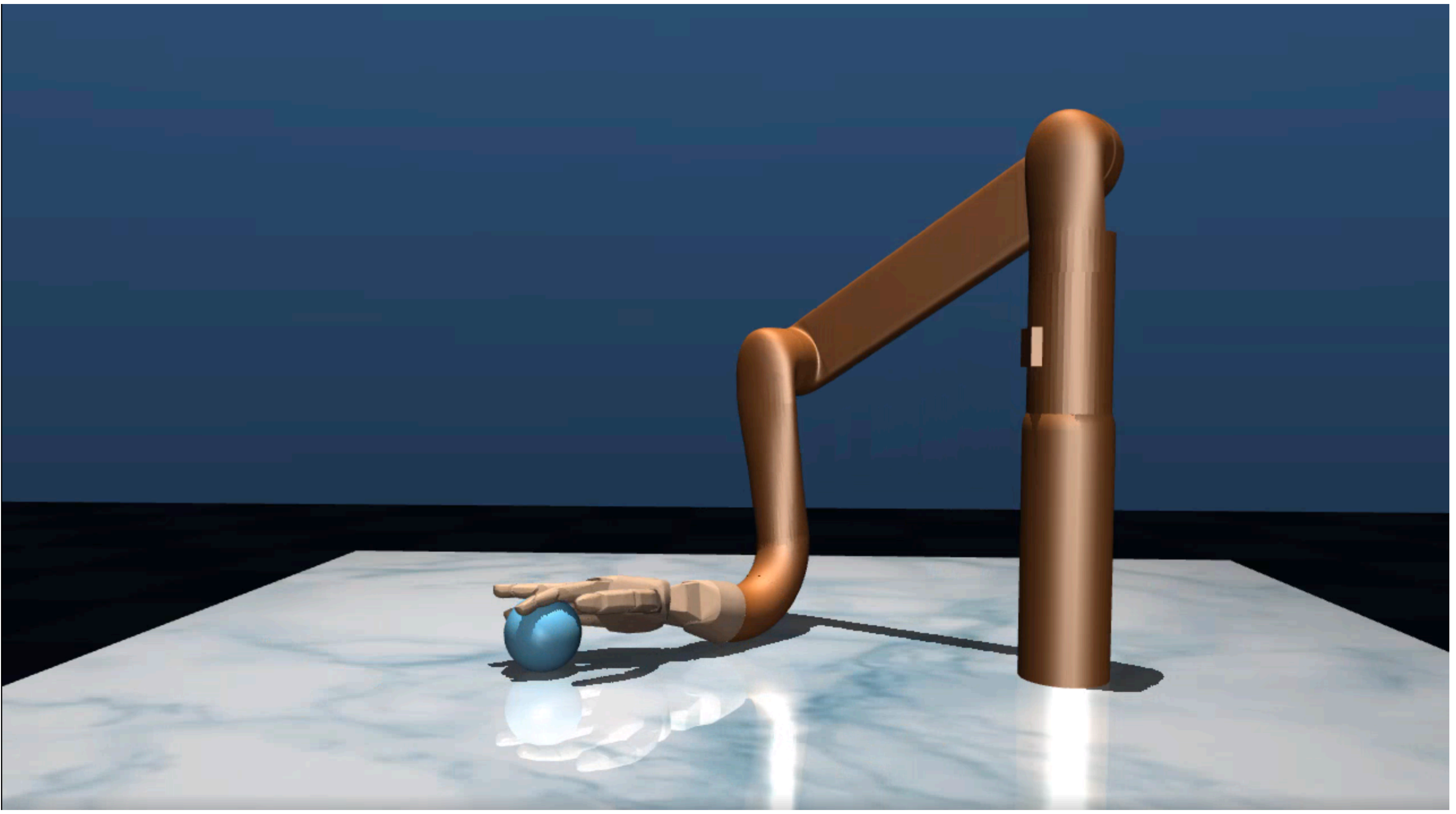

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$$\pi^\star = \arg\min_{\pi} \mathbb{E}\left[c(s_0, a_0) + c(s_1, a_1) + c(s_2, a_2) + \ldots c(s_{H-1}, a_{H-1}) \,\middle|\, s_0, \pi\right]$$

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  - The sampled trajectory is $\tau = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{H-1}, a_{H-1}, r_{H-1}\}$

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  - The rewards in this trajectory must be $r_t = r(s_t, a_t)$ (else $\rho_\pi(\tau) = 0$).

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  - For π stochastic:
    $\rho_\pi(\tau) = \mu(s_0)\pi(a_0 \mid s_0)P(s_1 \mid s_0, a_0)\ldots\pi(a_{H-2} \mid s_{H-2})P(s_{H-1} \mid s_{H-2}, a_{H-2})\pi(a_{H-1} \mid s_{H-1})$

The Probability of a Trajectory & The Objective

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- Objective: find policy π that maximizes our expected cumulative episodic reward:

$$\max_{\pi} \mathbb{E}_{\tau\sim\rho_\pi}\left[r(s_0, a_0) + r(s_1, a_1) + \ldots + r(s_{H-1}, a_{H-1})\right]$$

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  - Problem statement
  - Policy Evaluation

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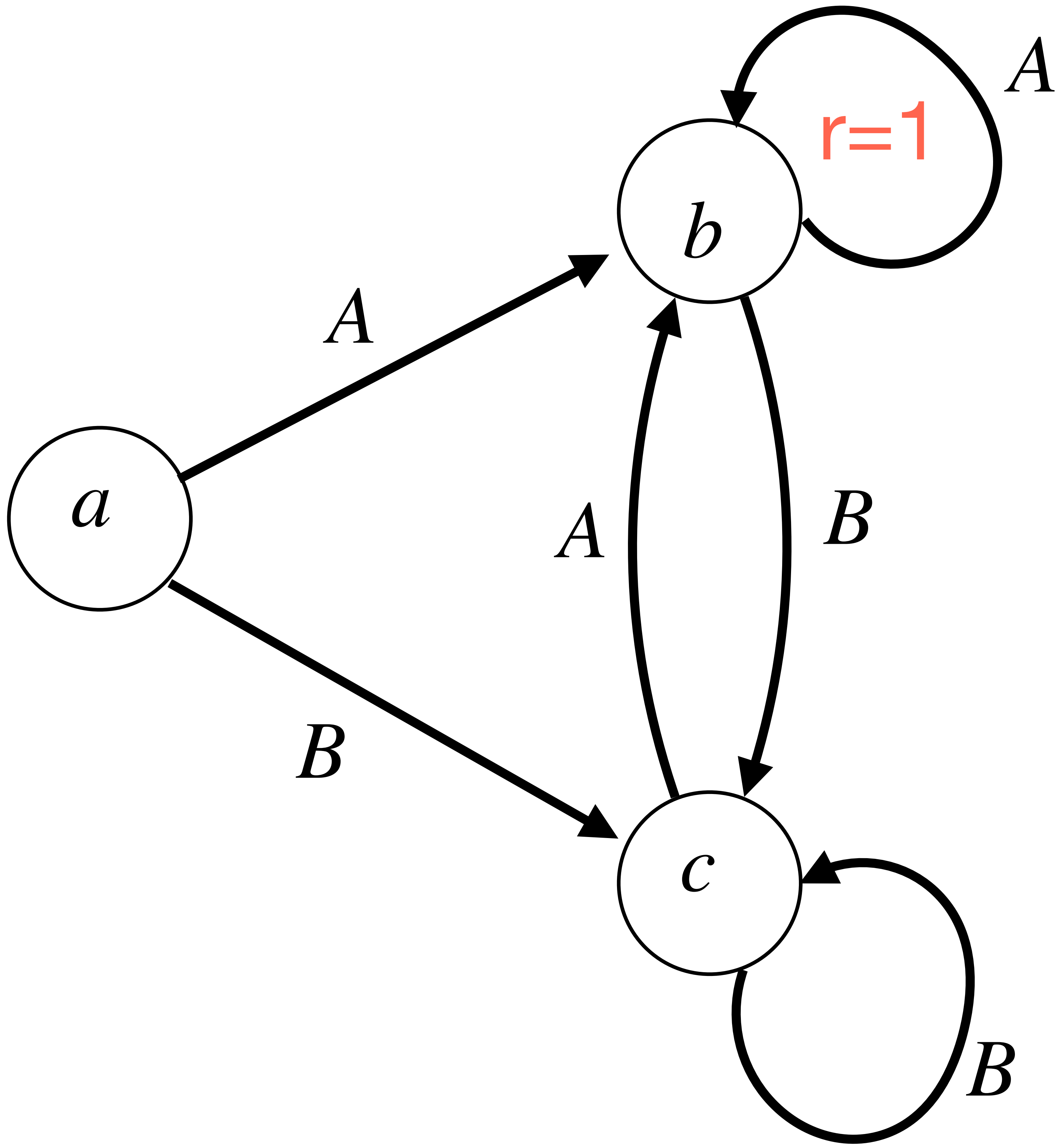
- At the last stage, for a stochastic policy,:

$$Q_{H-1}^\pi(s, a) = r(s, a) \qquad V_{H-1}^\pi(s) = \sum_a \pi_{H-1}(a \,|\, s) r(s, a)$$

Example of Policy Evaluation (i.e. computing V^π and Q^π)

Consider the following **deterministic** MDP w/ 3 states & 2 actions, with $H = 3$



Reward: $r(b, A) = 1$, & 0 everywhere else

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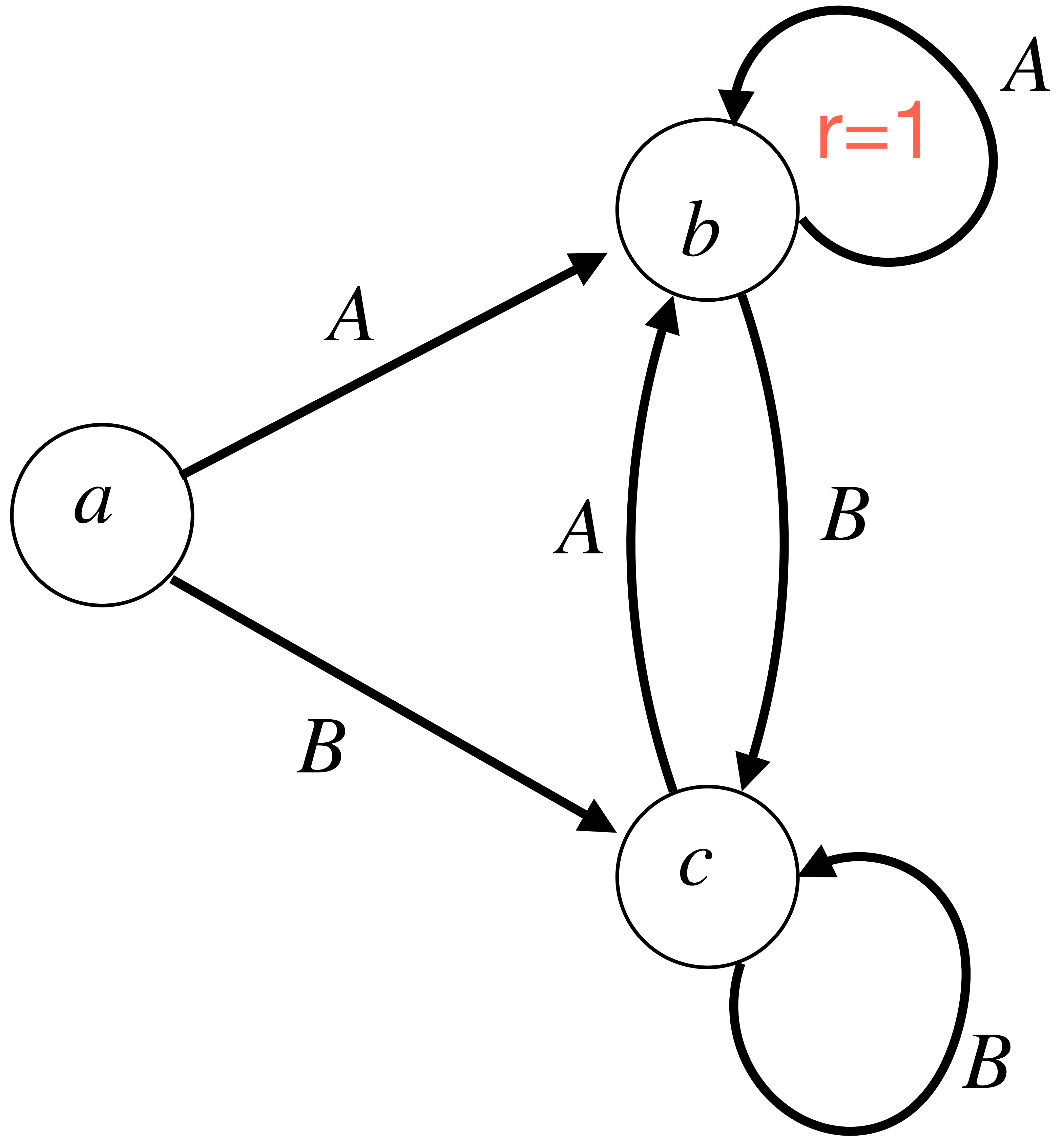

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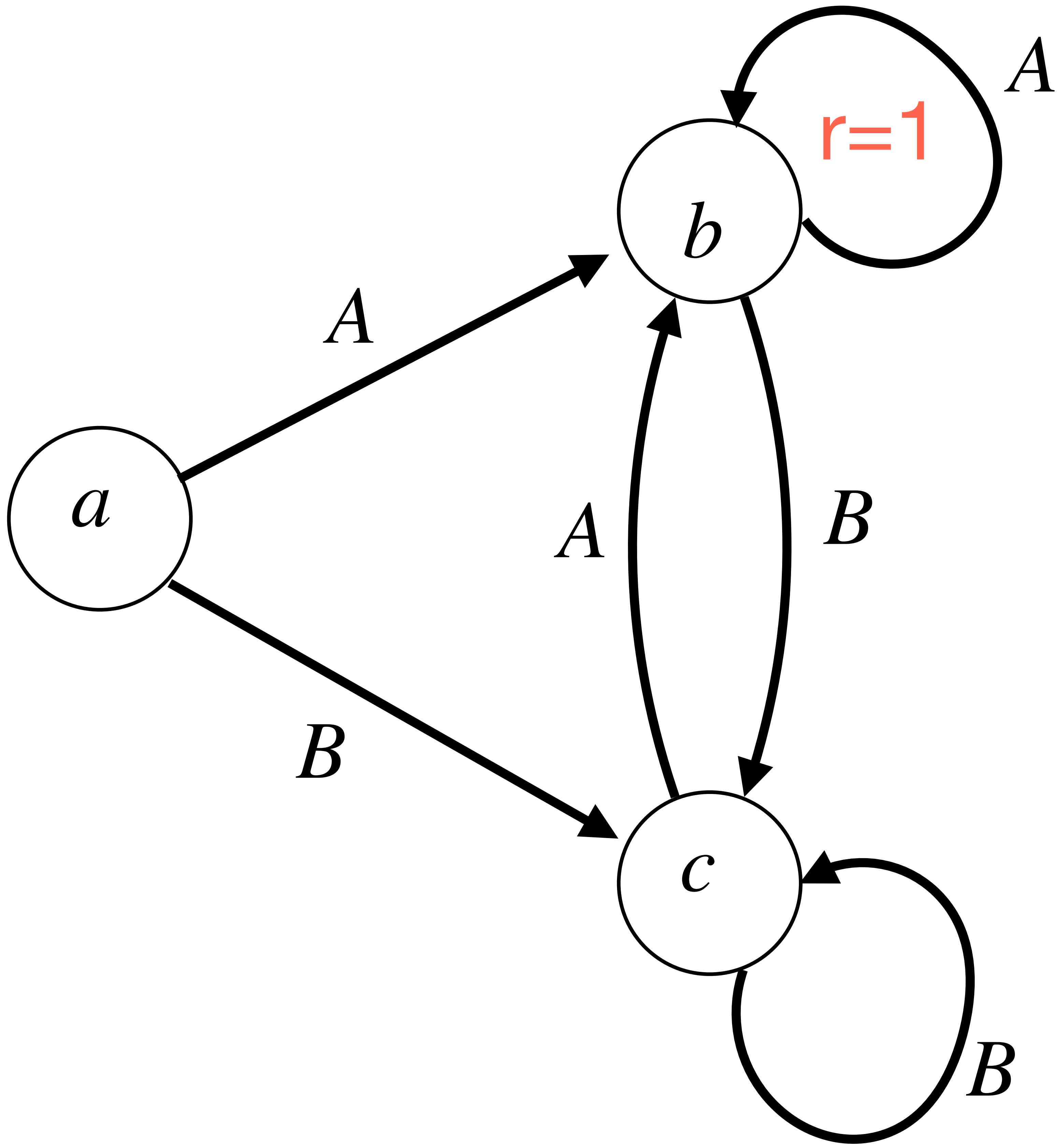

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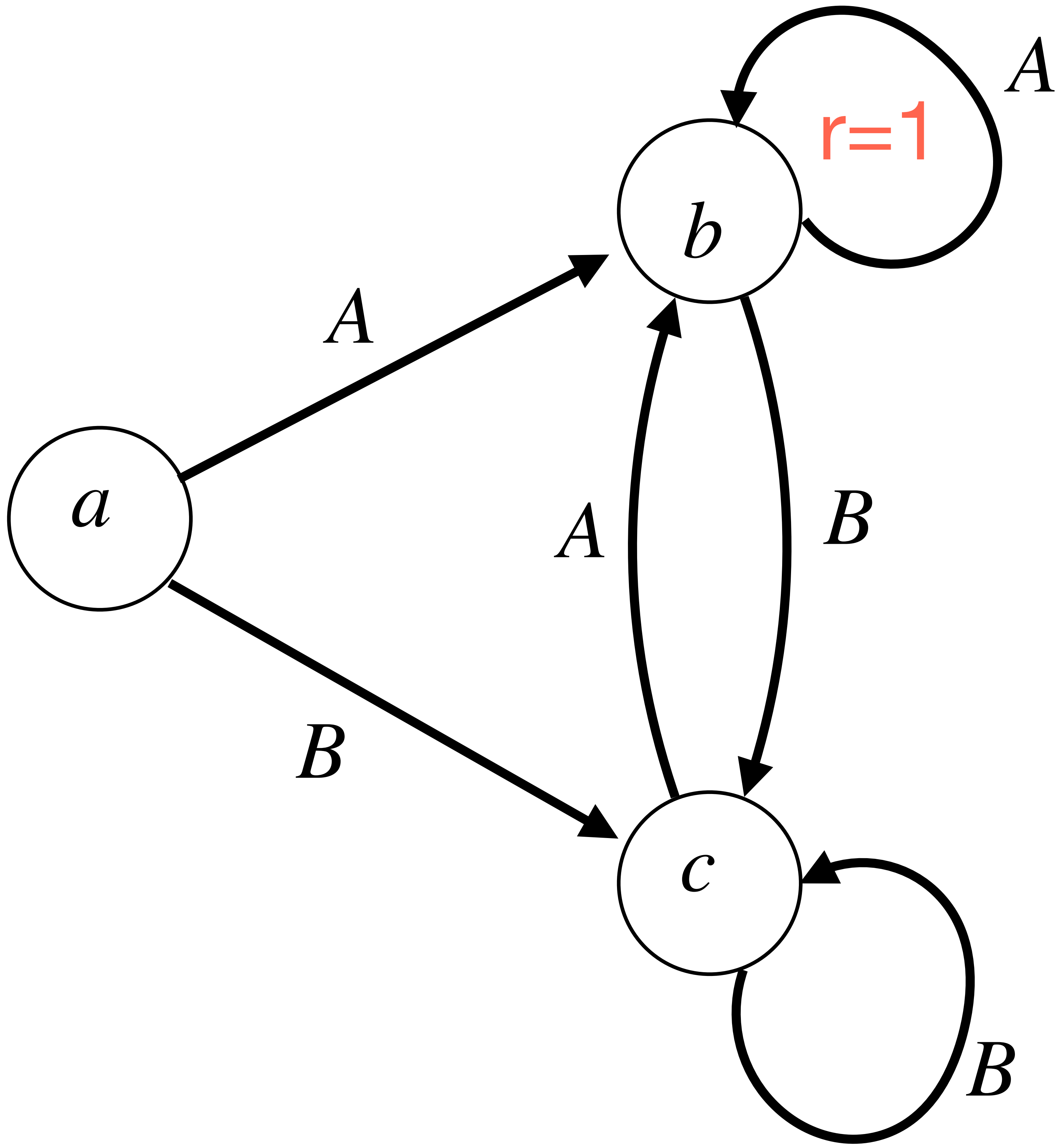

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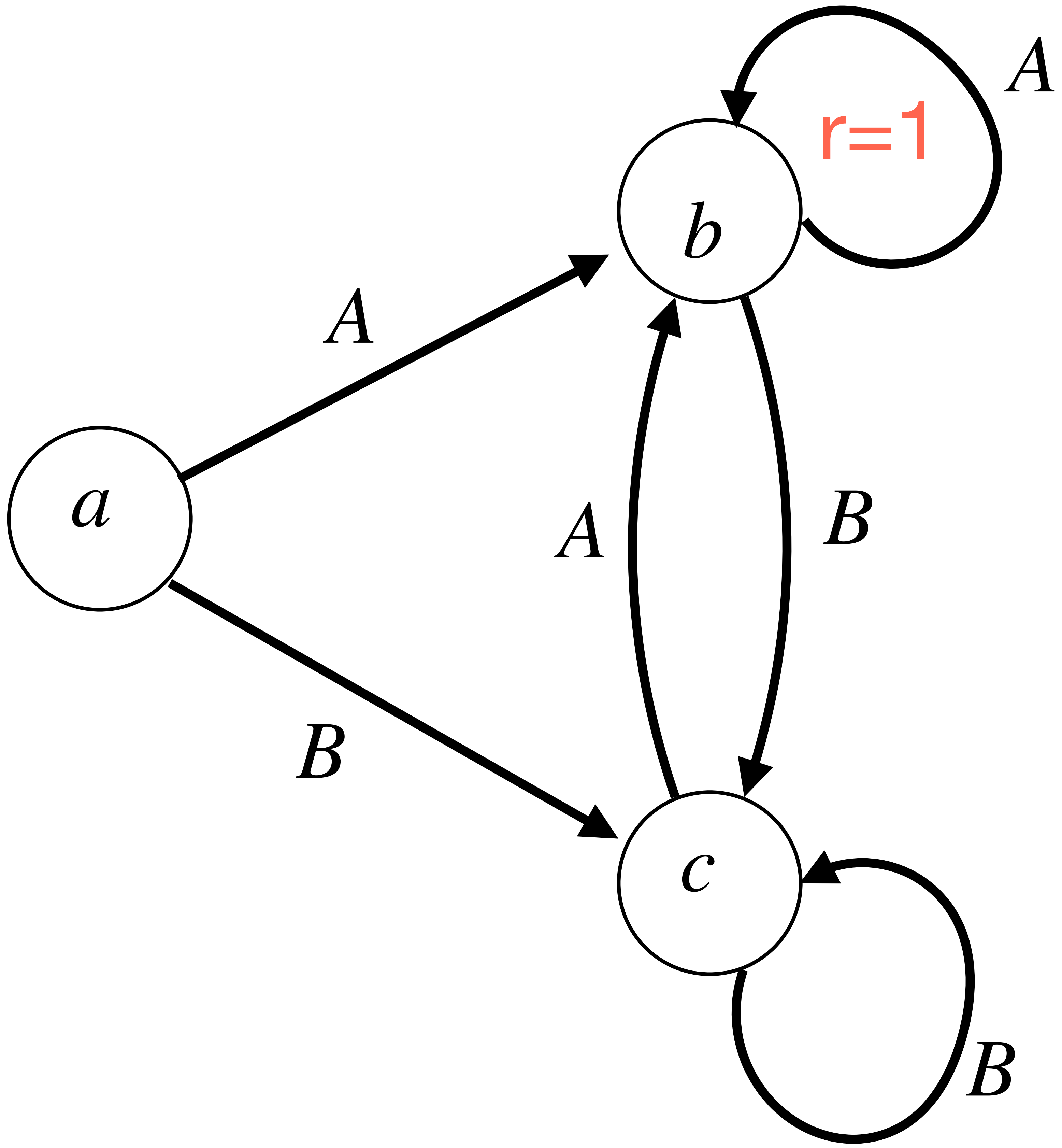

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- What is V^π?
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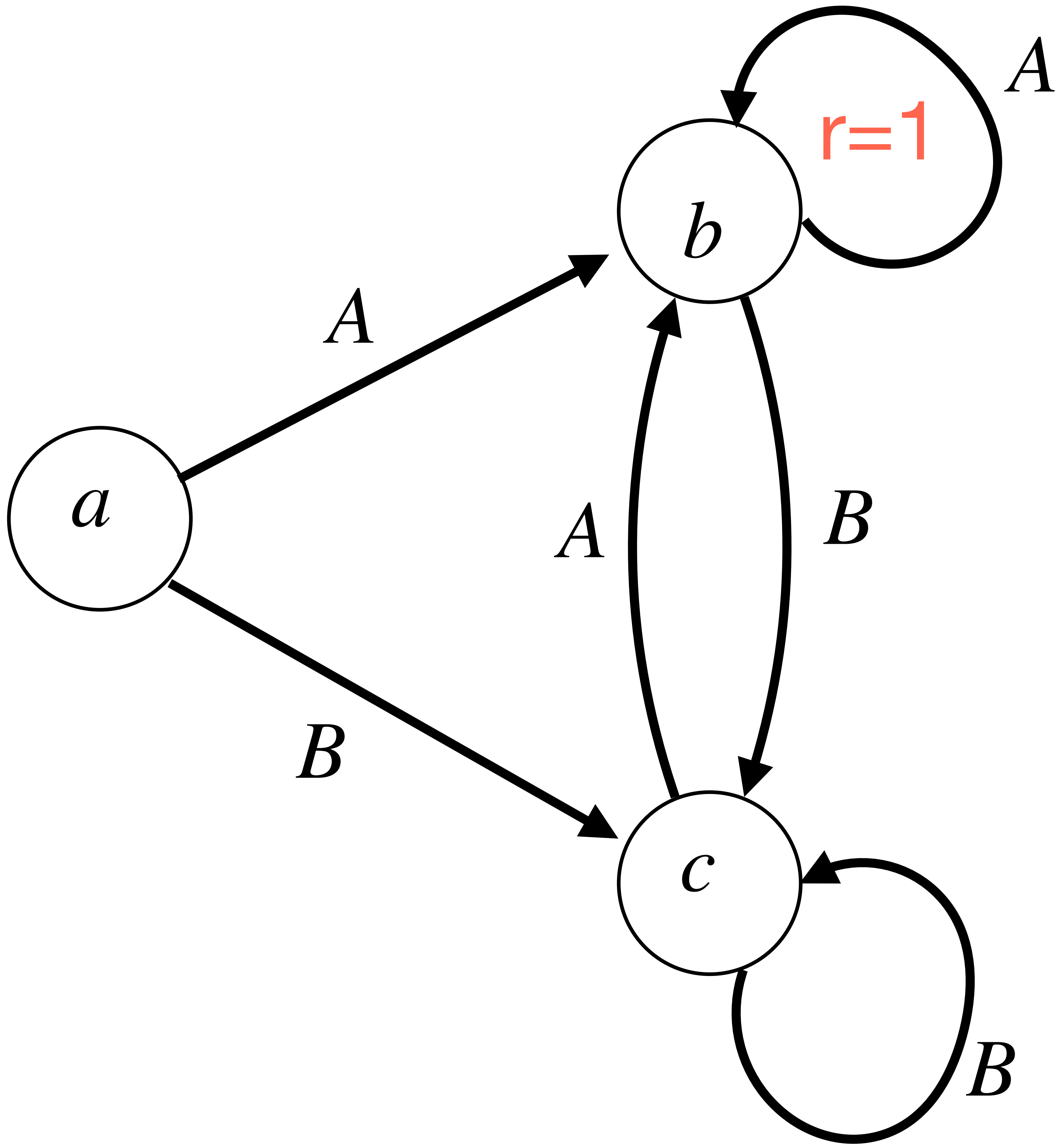

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  $V_0^\pi(a) = 1, V_0^\pi(b) = 2, V_0^\pi(c) = 1$

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Summary:

- **Finite horizon MDPs (a framework for RL):**
- Key concepts: **sampling a trajectory** $\rho_\pi(\tau)$, **V and Q functions**

Attendance:
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