PPO & Importance Sampling

Lucas Janson

CS/Stat 184(0): Introduction to Reinforcement Learning

Fall 2024

Today

- Feedback from last lecture
- Recap
- Importance Sampling (for PPO)
- PG review
- Exploration?

Feedback from feedback forms

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1. Thank you to everyone who filled out the forms!

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PG with a Learned Baseline:

Let $g'(\theta, \tau, b()) := \sum_{h=0}^{H-1} \nabla_\theta \ln \pi_\theta(a_h | s_h) \left(R_h(\tau) - b(s_h, h) \right)$

1. Initialize θ^0, parameters: $\eta^1, \eta^2, \ldots$
2. For $k = 0, \ldots$:
    1. Supervised Learning: Using N trajectories sampled under π_{θ^k}, estimate a baseline $\widetilde{b}$
       $\widetilde{b}(s, h) \approx V_h^{\theta^k}(s)$
    2. Obtain a trajectory $\tau \sim \rho_{\theta^k}$
       Compute $g'(\theta^k, \tau, \widetilde{b}())$
    3. Update: $\theta^{k+1} = \theta^k + \eta^k g'(\theta^k, \tau, \widetilde{b}())$

Note that regardless of our choice of $\widetilde{b}$, we still get unbiased gradient estimates.

The Performance Difference Lemma (PDL)

- Let $\rho_{\widetilde{\pi},s}$ be the distribution of trajectories from starting state s acting under $\widetilde{\pi}$. (we are making the starting distribution explicit now).
- For any two policies π and $\widetilde{\pi}$ and any state s,

$$V^{\widetilde{\pi}}(s) - V^{\pi}(s) = \mathbb{E}_{\tau \sim \rho_{\widetilde{\pi},s}} \left[\sum_{h=0}^{H-1} A^{\pi}(s_h, a_h, h) \right]$$

Comments:

- Helps us think about error analysis, instabilities of fitted PI, and sub-optimality.
- Helps to understand algorithm design (TRPO, NPG, PPO)
- This also motivates the use of “local” methods (e.g. policy gradient descent)

Trust Region Policy Optimization (TRPO)

1. Initialize θ^0
2. For $k = 0,\ldots,K$:

   try to approximately solve:

$$\theta^{k+1} = \arg\max_{\theta} \mathbb{E}_{s_0,\ldots,s_{H-1}\sim\rho_{\pi_{\theta^k}}} \left[\sum_{h=0}^{H-1} \mathbb{E}_{a_h\sim\pi_\theta(\cdot|s_h)} \left[A^{\pi_{\theta^k}}(s_h, a_h, h)\right] \right]$$

$$\text{s.t. } KL\left(\rho_{\pi_{\theta^k}} | \rho_{\pi_\theta}\right) \leq \delta$$

3. Return π_{θ^K}

NPG has a closed form update!

1. Initialize θ^0
2. For $k = 0, \ldots, K$:

$$\theta^{k+1} = \arg\max_{\theta} \nabla_\theta J(\theta^k)^\top (\theta - \theta^k)$$

$$\text{s.t. } (\theta - \theta^k)^\top F_{\theta^k} (\theta - \theta^k) \le \delta$$

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Linear objective and quadratic convex constraint: we can solve it optimally!

Indeed this gives us:

$$\theta^{k+1} = \theta^k + \eta F_{\theta^k}^{-1} \nabla_\theta J(\theta^k)$$

Where $\eta = \sqrt{\dfrac{\delta}{\nabla_\theta J(\theta^k)^\top F_{\theta^k}^{-1} \nabla_\theta J(\theta^k)}}$

Proximal Policy Optimization (PPO)

1. Initialize θ^0
2. For $k = 0,\ldots,K$:
   use SGD to approximately solve:
   $\theta^{k+1} = \arg\max_{\theta} \ell^k(\theta)$
   where:
   $$\ell^k(\theta) := \mathbb{E}_{s_0,\ldots,s_{H-1}\sim\rho_{\pi_{\theta^k}}}\left[\sum_{h=0}^{H-1}\mathbb{E}_{a_h\sim\pi_\theta(\cdot|s_h)}\left[A^{\pi_{\theta^k}}(s_h, a_h, h)\right]\right] - \lambda\mathbb{E}_{\tau\sim\rho_{\pi_{\theta^k}}}\left[\sum_{h=0}^{H-1}\ln\frac{1}{\pi_\theta(a_h\,|\,s_h)}\right]$$
3. Return π_{θ^K}

How do we estimate this objective?

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SGD and Importance Sampling

- Recall that SGD requires an unbiased estimate of the objective function's gradient
- This was easy when the objective function was an expectation, and the only θ-dependence appears inside the expectation
  - This was true for supervised learning / ERM
  - Not true for RL, and was part of why we needed likelihood ratio method in REINFORCE
- When not true (as in PPO), we want to make it so, if possible
- Enter: importance sampling
  - rewrites expectations by changing the distribution the expectation is over
  - we will use this to move that distribution's θ-dependence inside the expectation
- **Key point**: once all θ-dependence inside objective's expectation,
  - Can estimate objective unbiasedly via sample average
  - Can estimate objective's gradient unbiasedly via gradient of sample average

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- Assume: we have an (i.i.d.) dataset $x_1, \dots x_N$, where $x_i \sim p$, where p is known, and
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- What about the variance of this estimator?

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- we will use importance sampling:

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2. Obtain M NEW trajectories $\tau_1, \ldots \tau_M \sim \rho_{\pi_{\theta^k}}$

   Set $\widehat{\ell}^k(\theta) = \dfrac{1}{M} \sum_{m=1}^{M} \sum_{h=0}^{H-1} \left(\dfrac{\pi_\theta(a_h^m \mid s_h^m)}{\pi_{\theta^k}(a_h^m \mid s_h^m)} \left(R_h(\tau_m) - \widetilde{b}(s_h^m, h) \right) - \lambda \ln \dfrac{1}{\pi_\theta(a_h^m \mid s_h^m)} \right)$

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$g(\theta^k)$ is unbiased for $\nabla_\theta \ell^k(\theta)\Big|_{\theta=\theta^k}$

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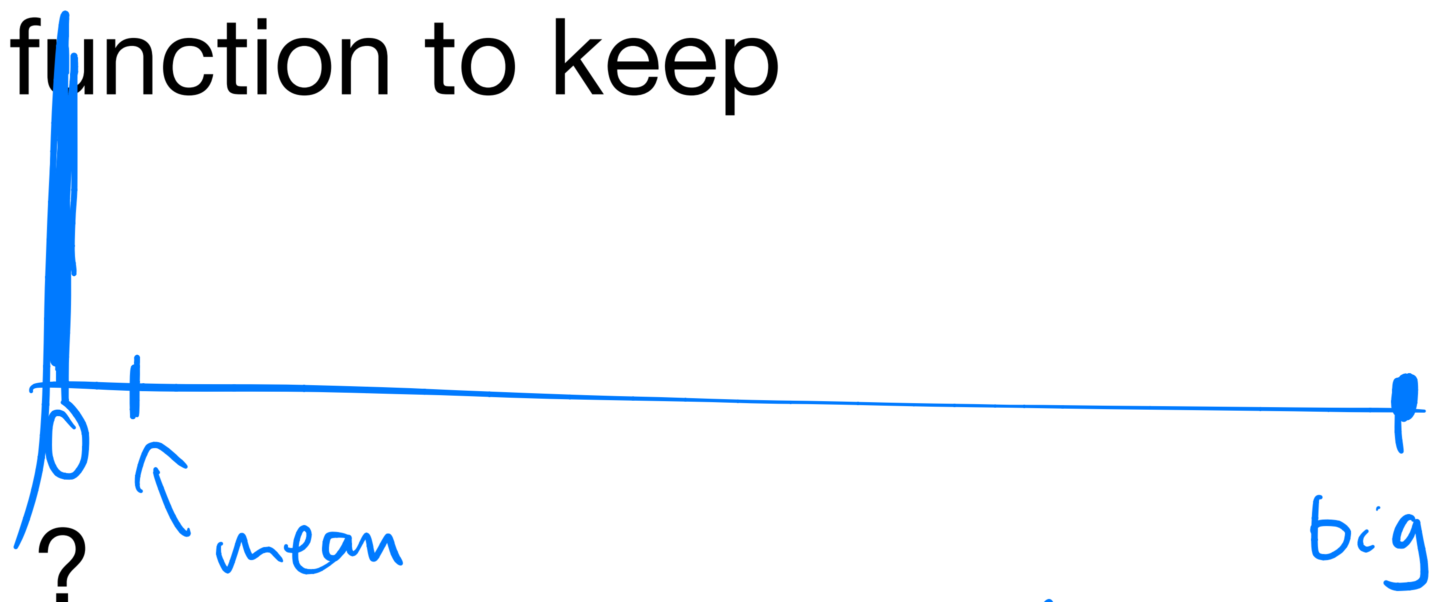

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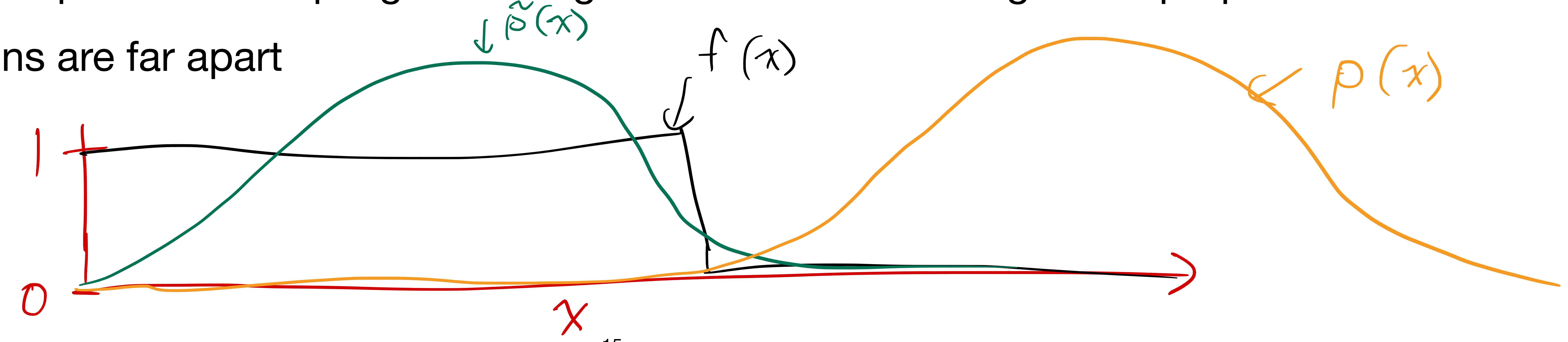
$\mathbb{E}_{x\sim p(x)}\left[\overbrace{\frac{\tilde{p}(x)}{p(x)} f(x)}^{Y}\right]$

- Or in PG, why do we sample online, when the likelihood ratio method still gives unbiasedness for trajectories sampled from π_{θ^0}?

- **Variance**: Importance sampling incurs big variance when the target and proposal distributions are far apart
- Picture:

$\mathbb{E}_{x\sim \tilde{p}(x)}\left[f(x)\right] = 1$



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- Really not so different, and NPG provides a unifying perspective: TRPO/PPO essentially doing PG with a 2nd-order correction to the gradient

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**Trust Region Policy
Optimization (TRPO)**

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Trust Region Policy Optimization (TRPO)

Approximation
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Natural Policy Gradient (NPG)

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PPO gets 2nd-order optimization benefits over PG and 1st-order computation benefits over TRPO/NPG

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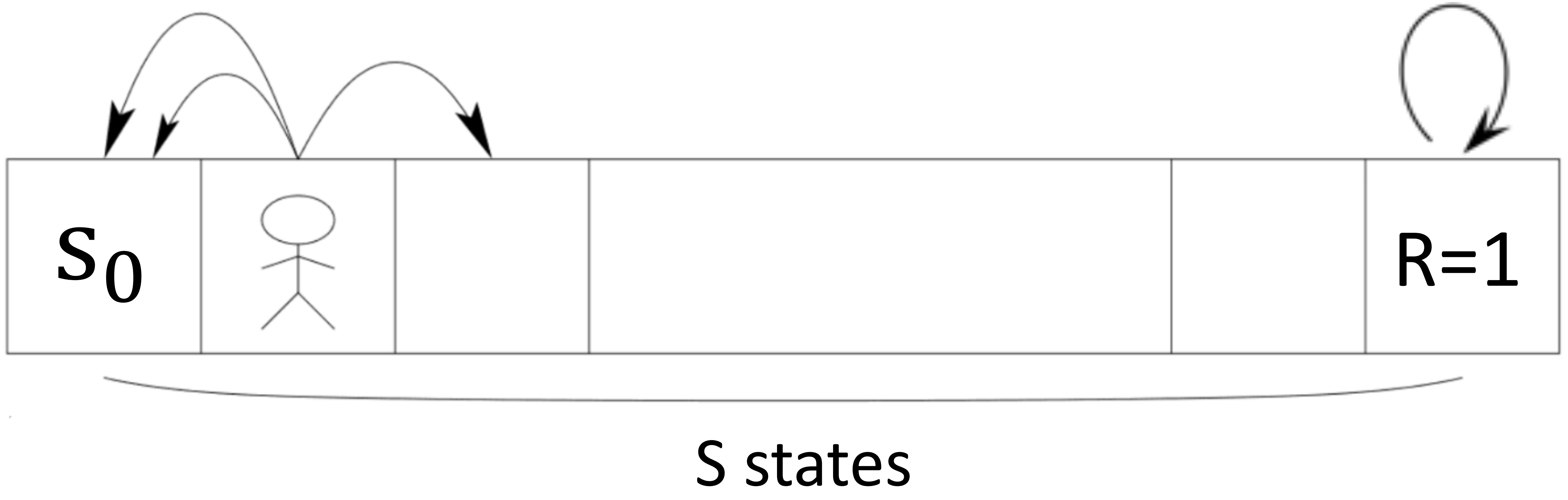
“Lack of Exploration” leads to Optimization and Statistical Challenges



Thrun ‘92

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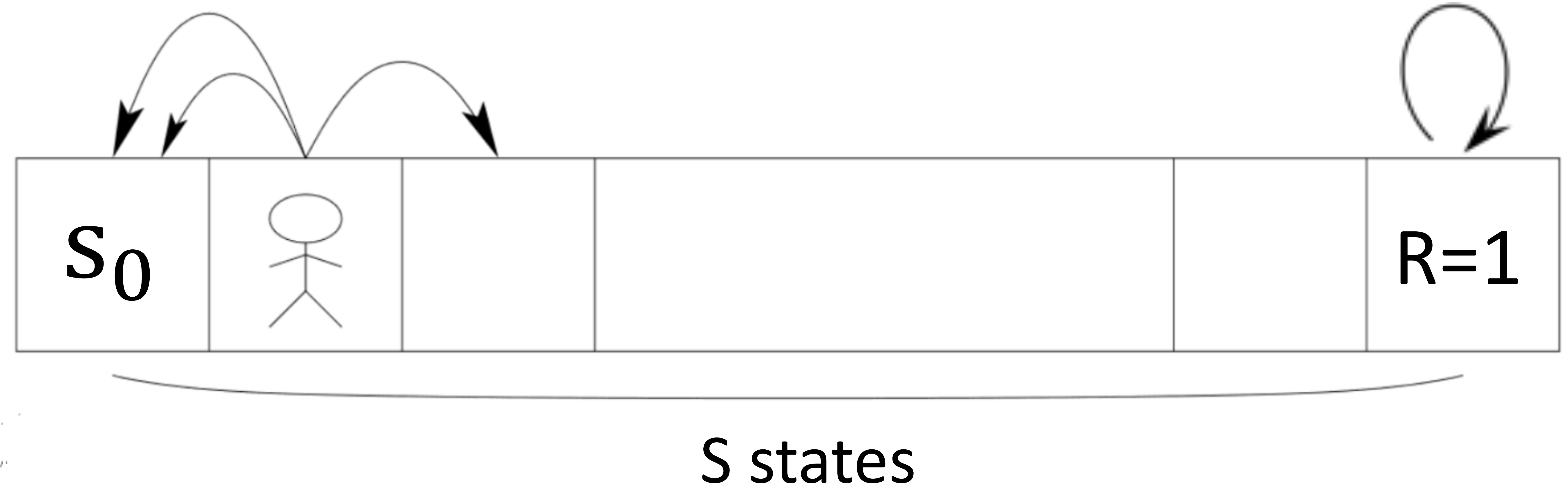

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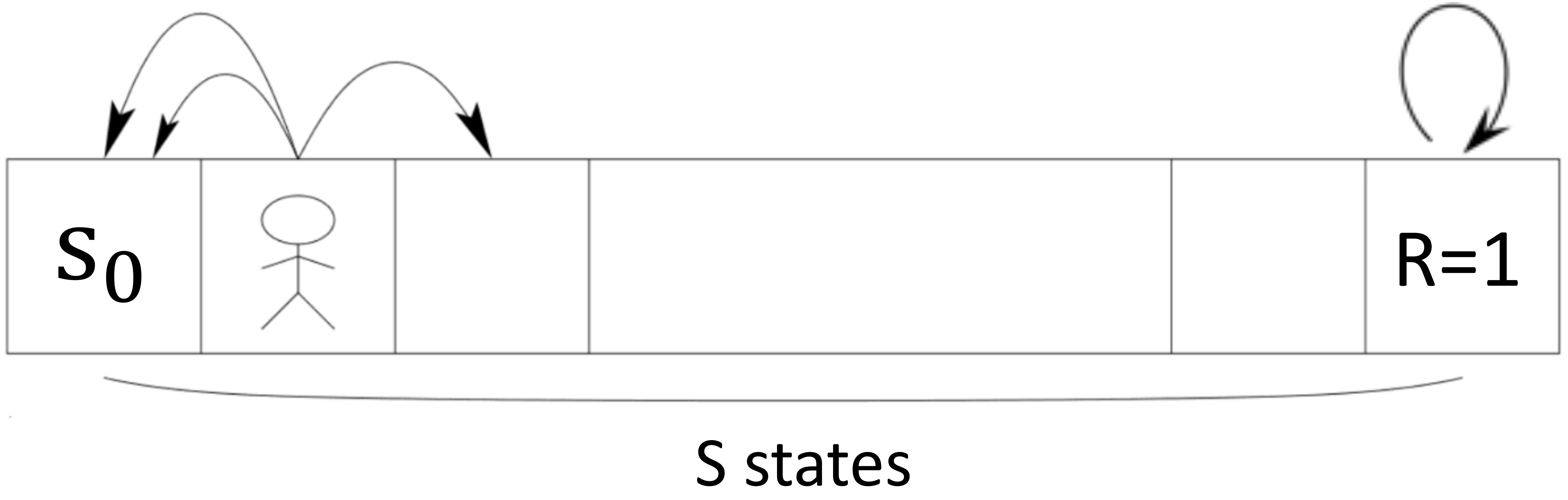

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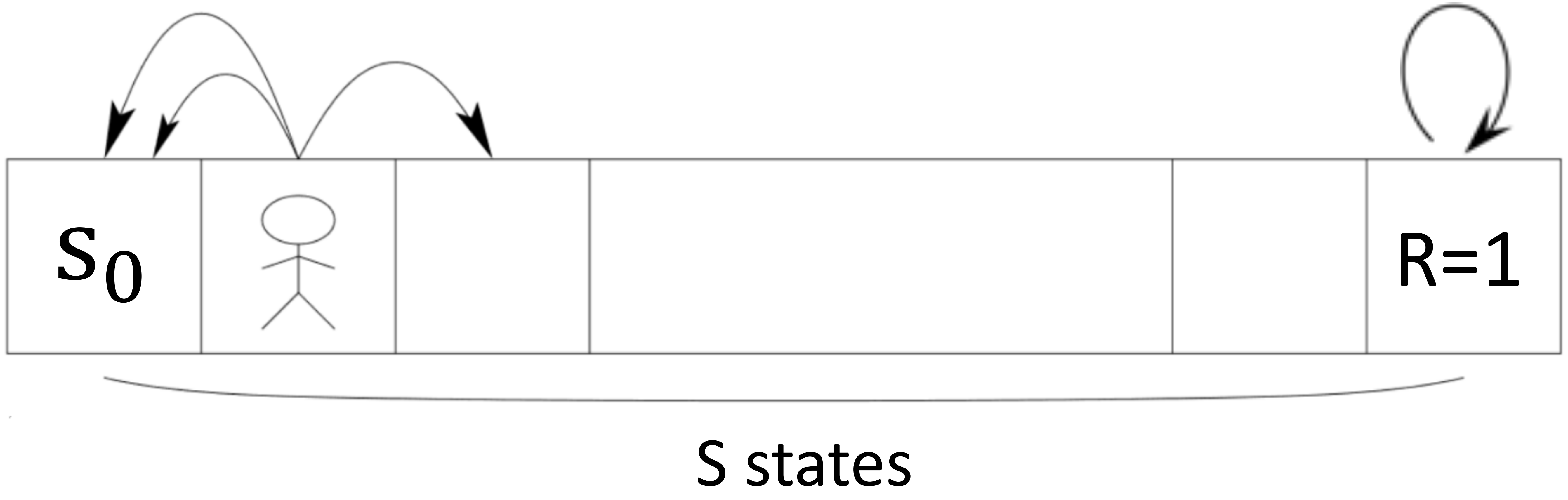

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  - Holds for (sample based) Fitted DP
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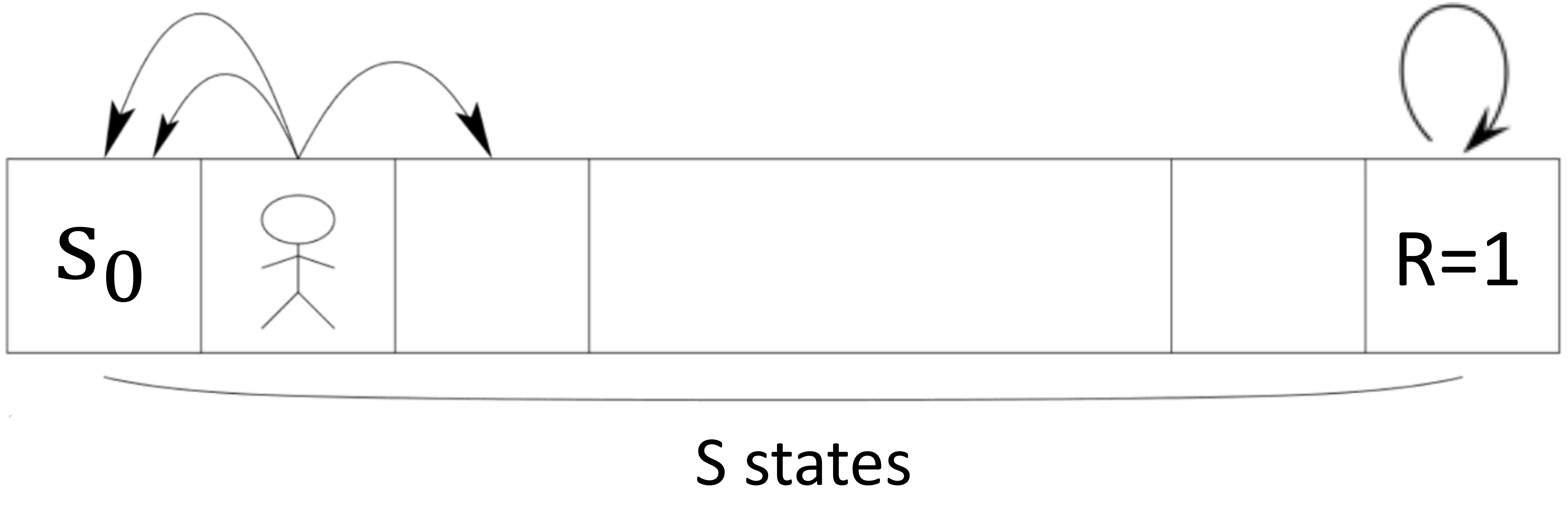

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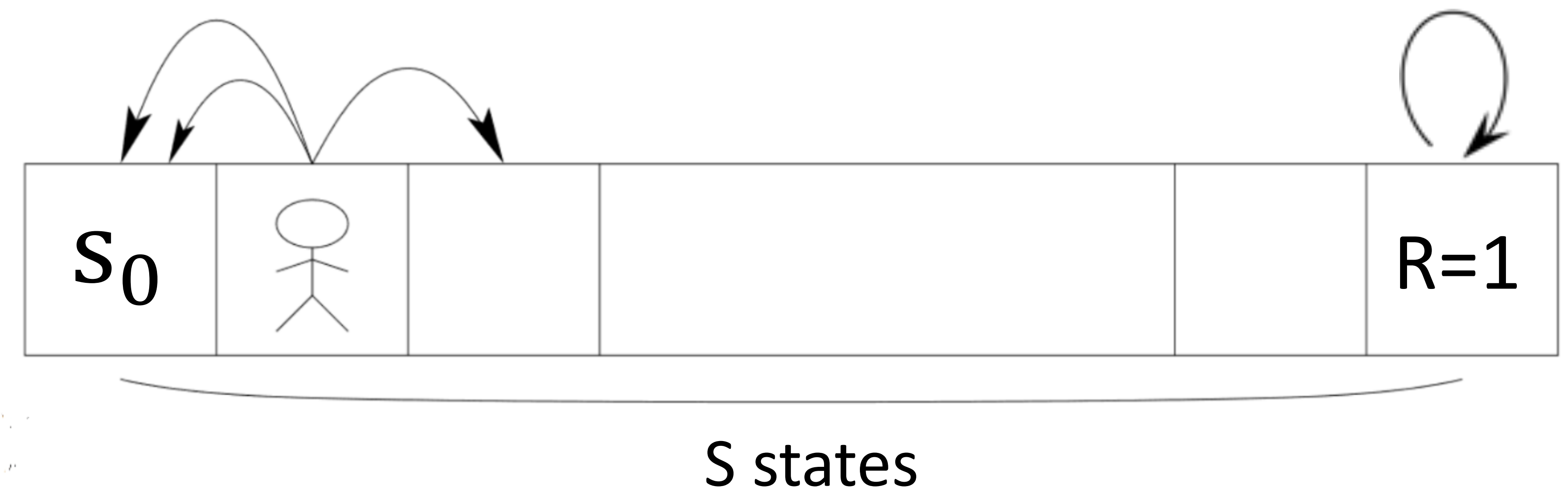
- Basically, for these approaches, there is no hope of learning the optimal policy if $\mu(s_0) = 1$.

Let's examine the role of μ



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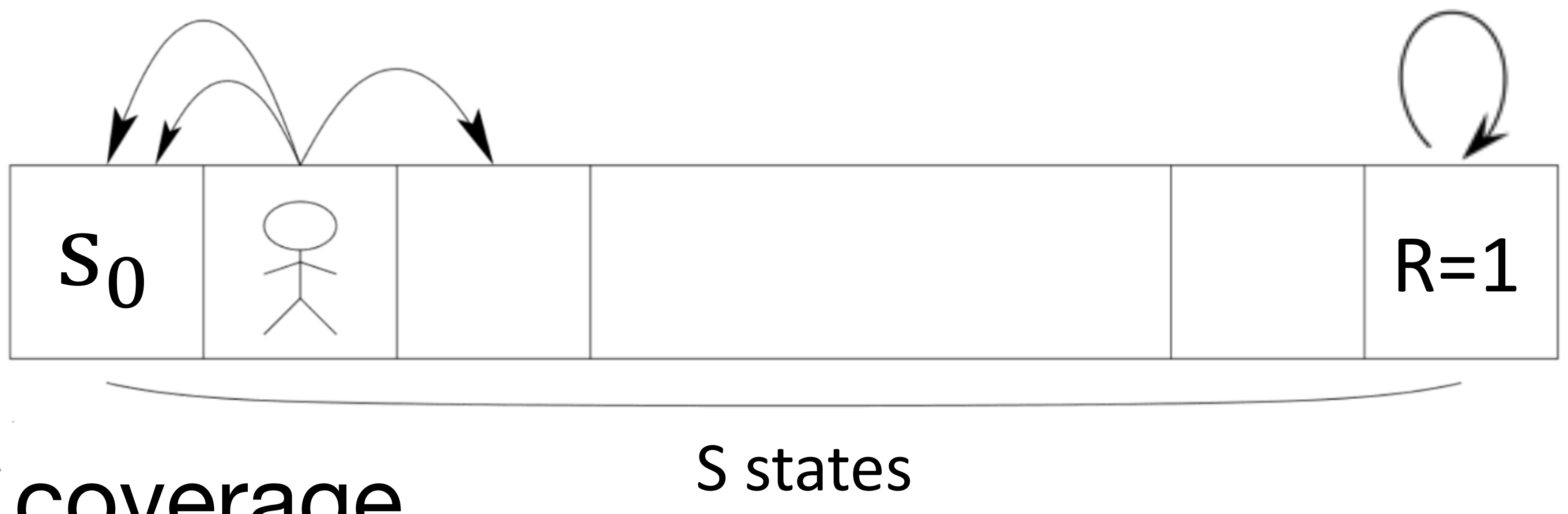

Thrun '92

- Suppose that somehow the distribution μ had better coverage.
  - e.g, if μ was uniform overall states in our toy problem, then all approaches we covered would work (with mild assumptions)
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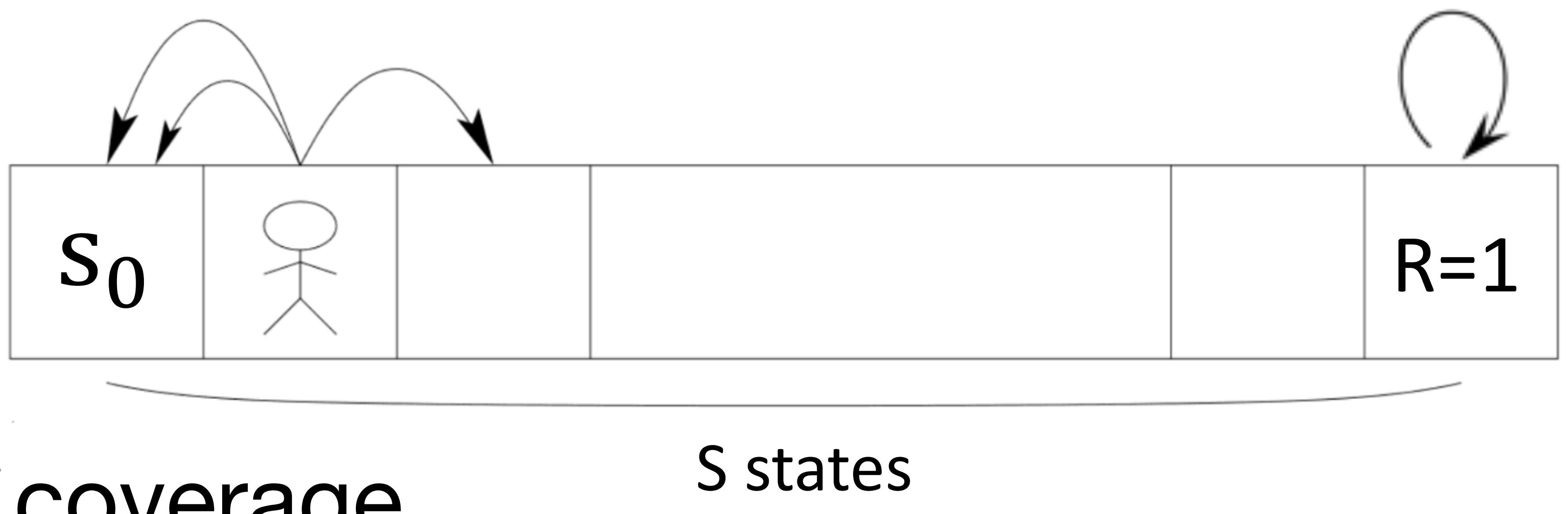

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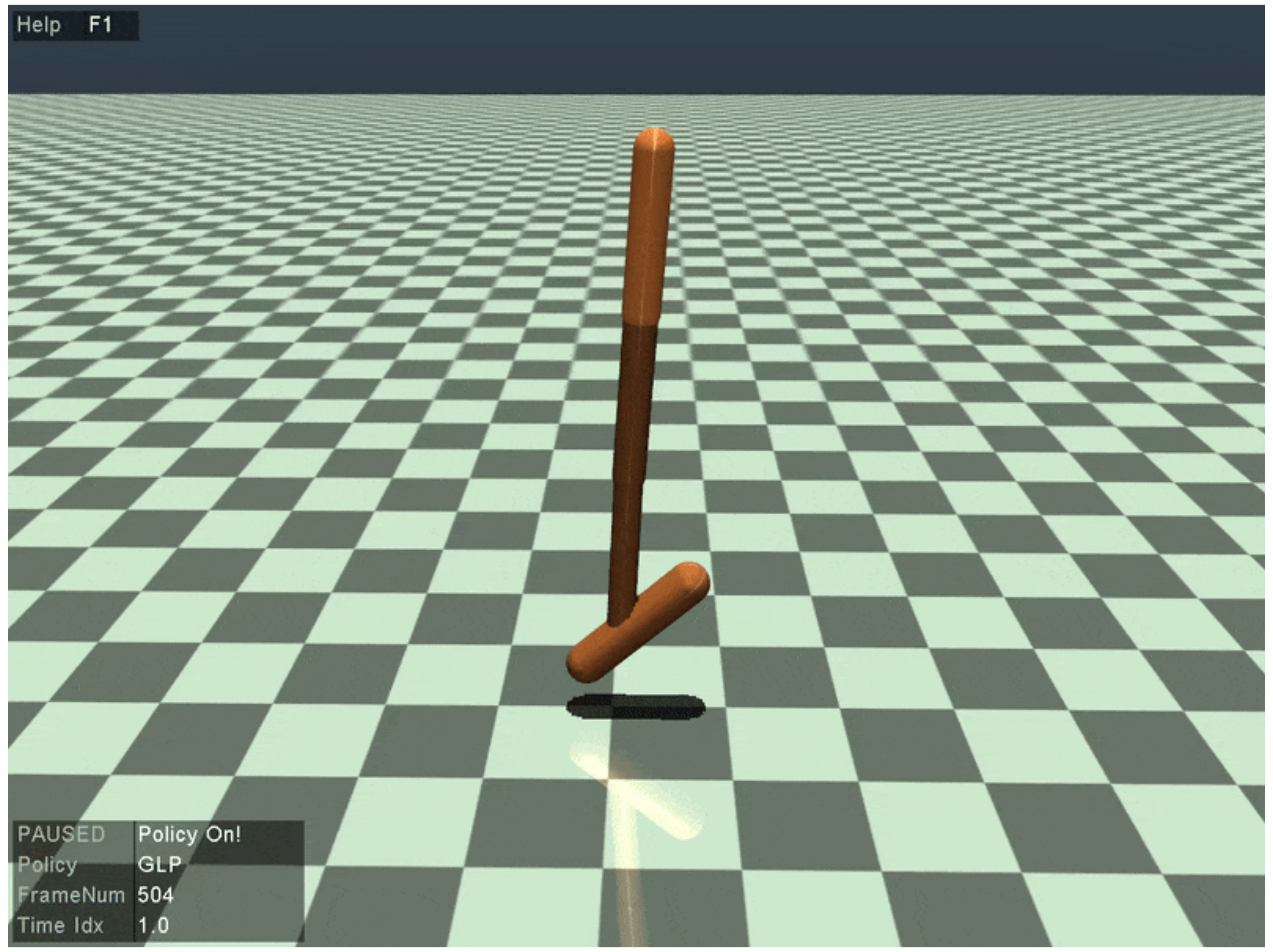
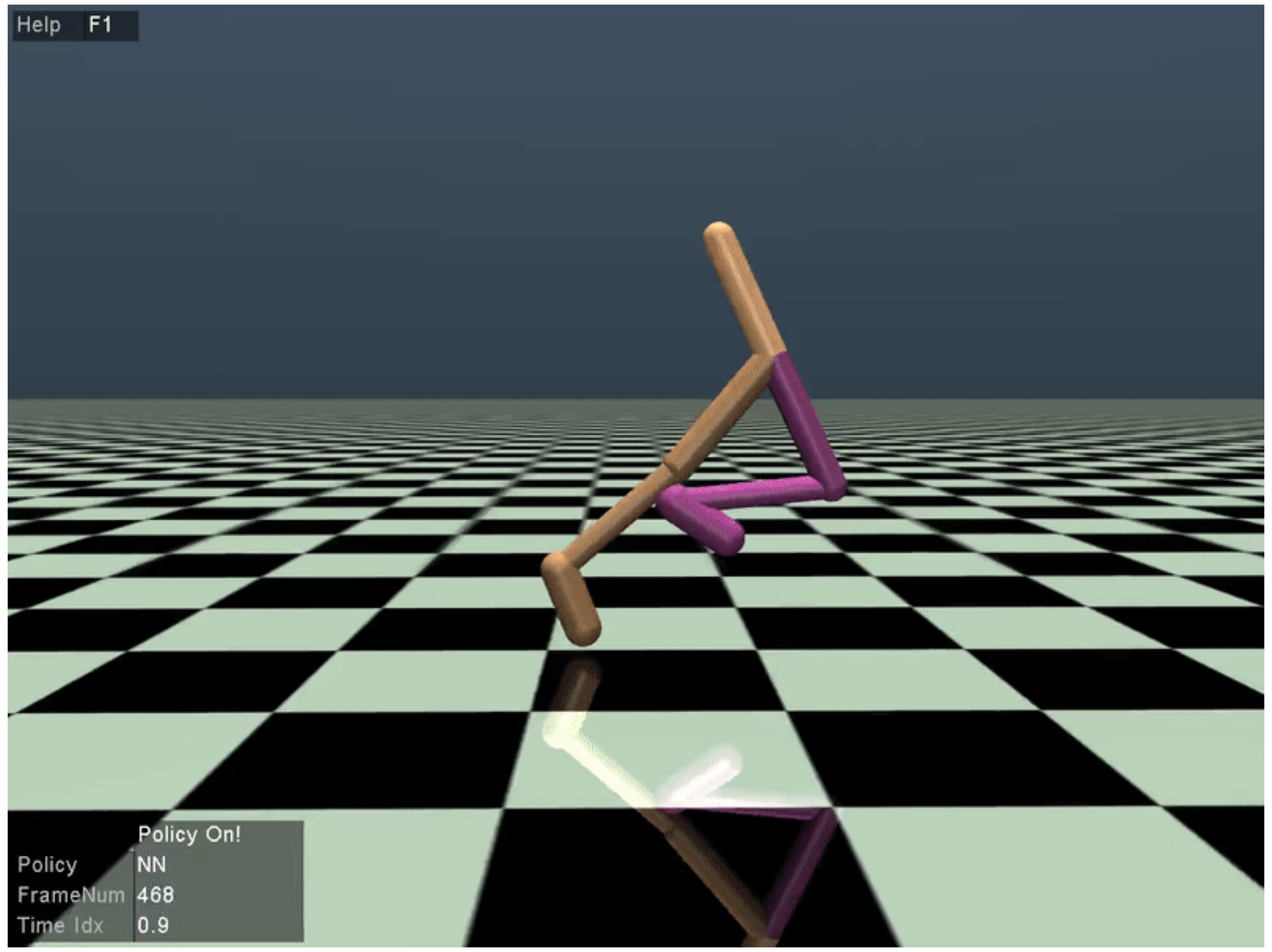
  - Theory: TRPO/NPG/PPO have better guarantees than fitted DP methods (assuming some "coverage")
- Strategies without coverage:
  - If we have a simulator, sometimes we can design μ to have better coverage.
    - this is helpful for robustness as well.
  - Imitation learning (next time).
    - An expert gives us samples from a "good" μ.
  - Explicit exploration:
    - UCB-VI: we'll merge two good ideas!
    - Encourage exploration in PG methods.
  - Try with reward shaping

Aside: Brittle policies if we train starting from only from one configuration!





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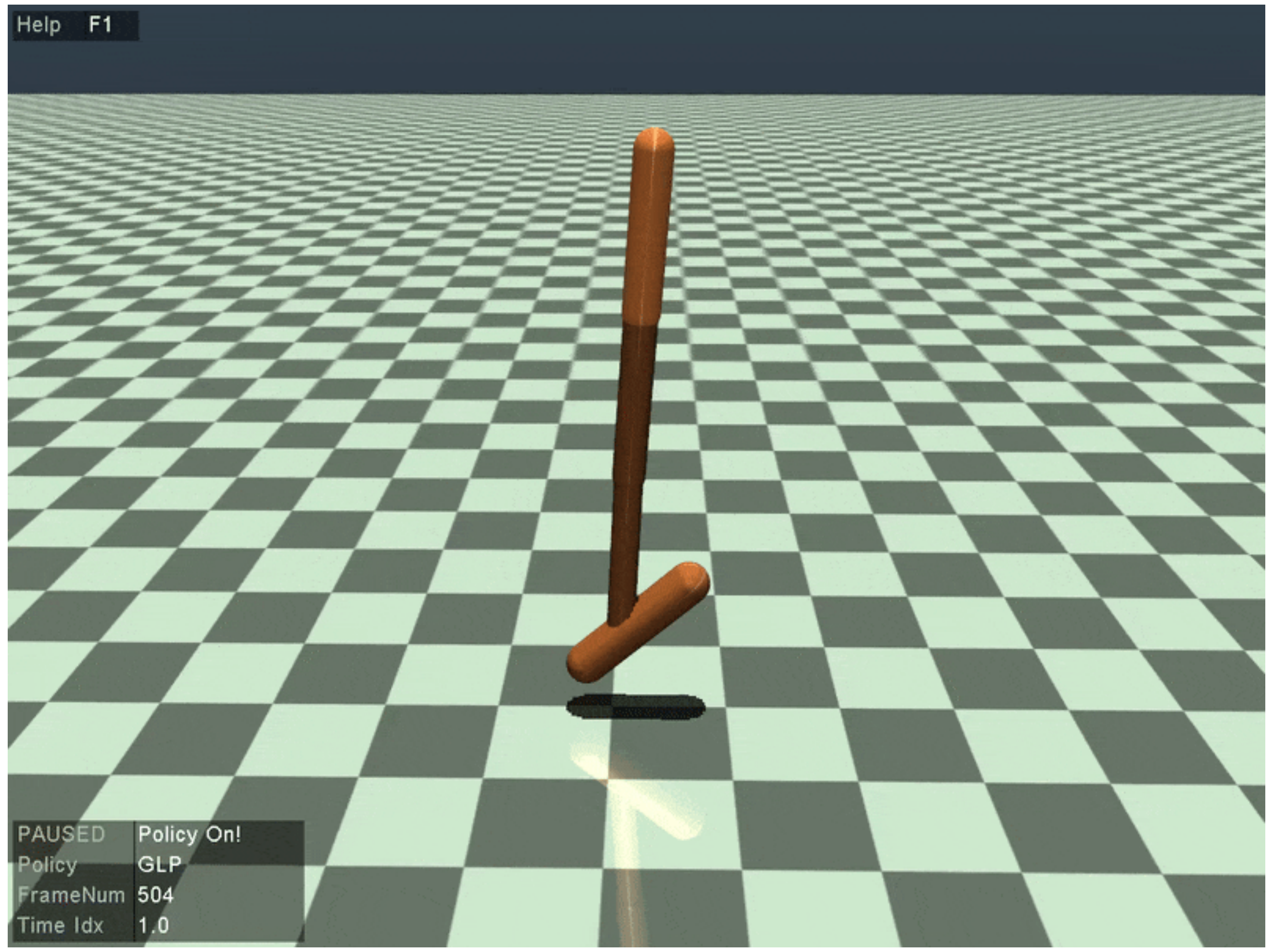

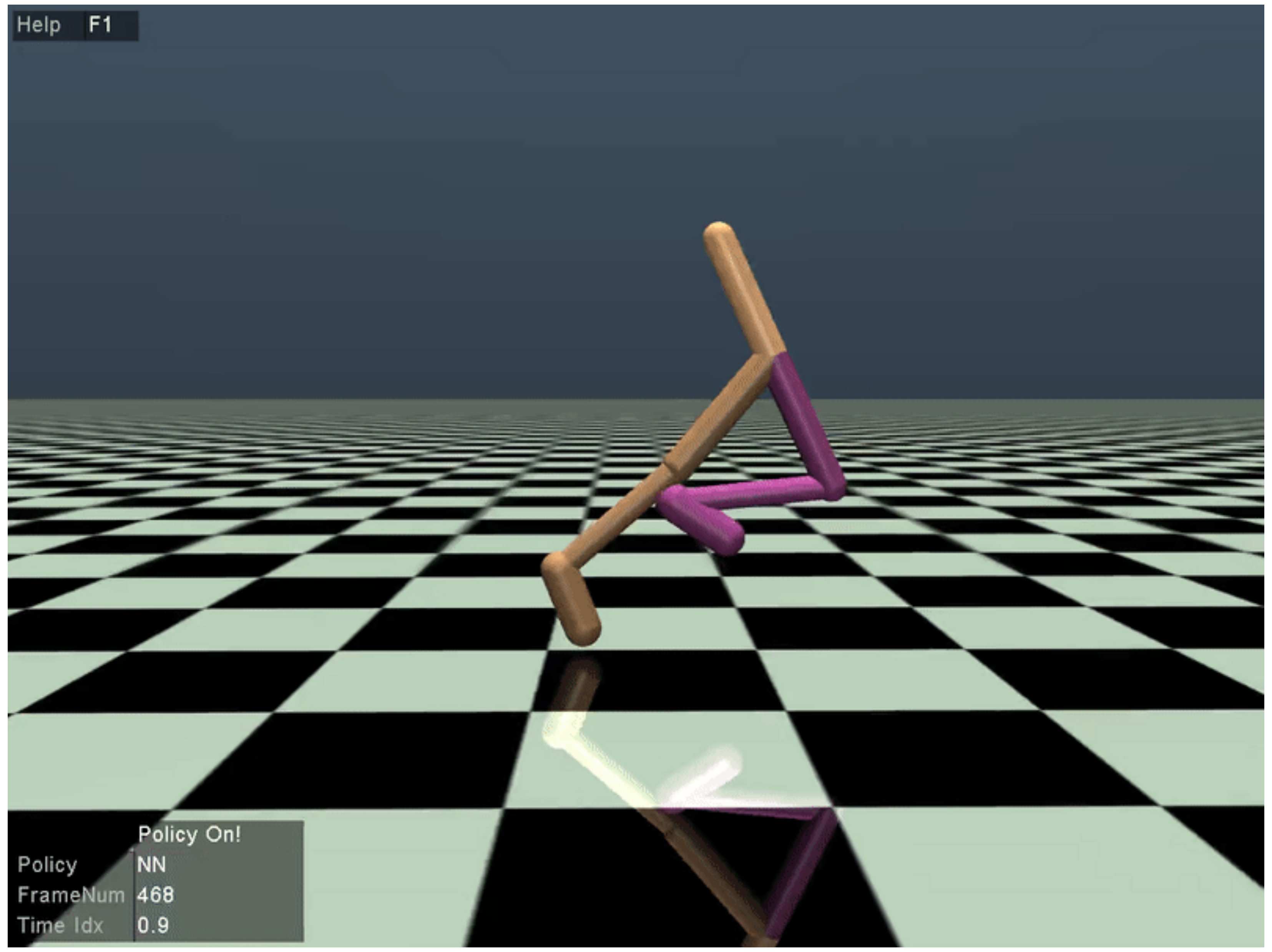

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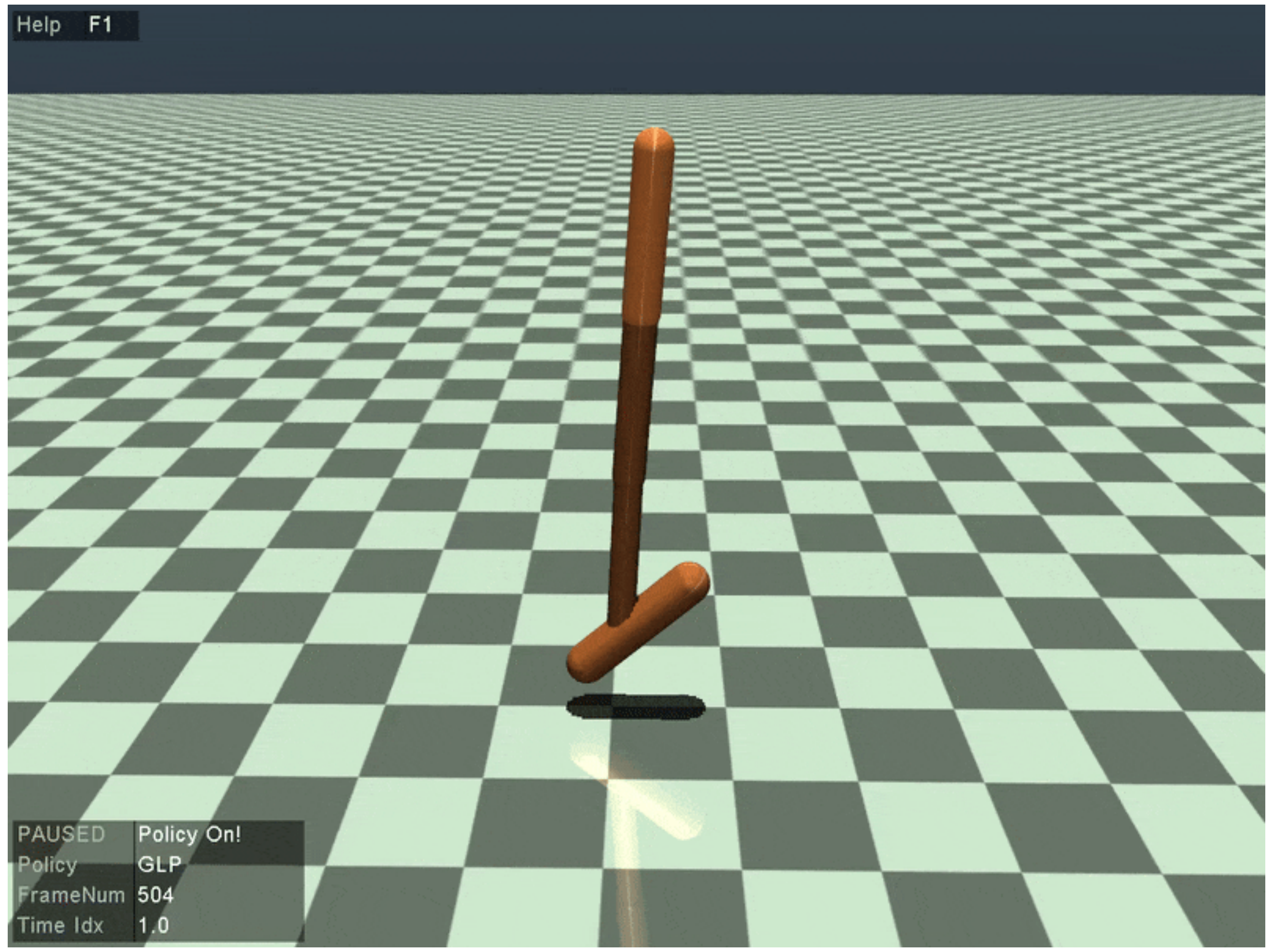

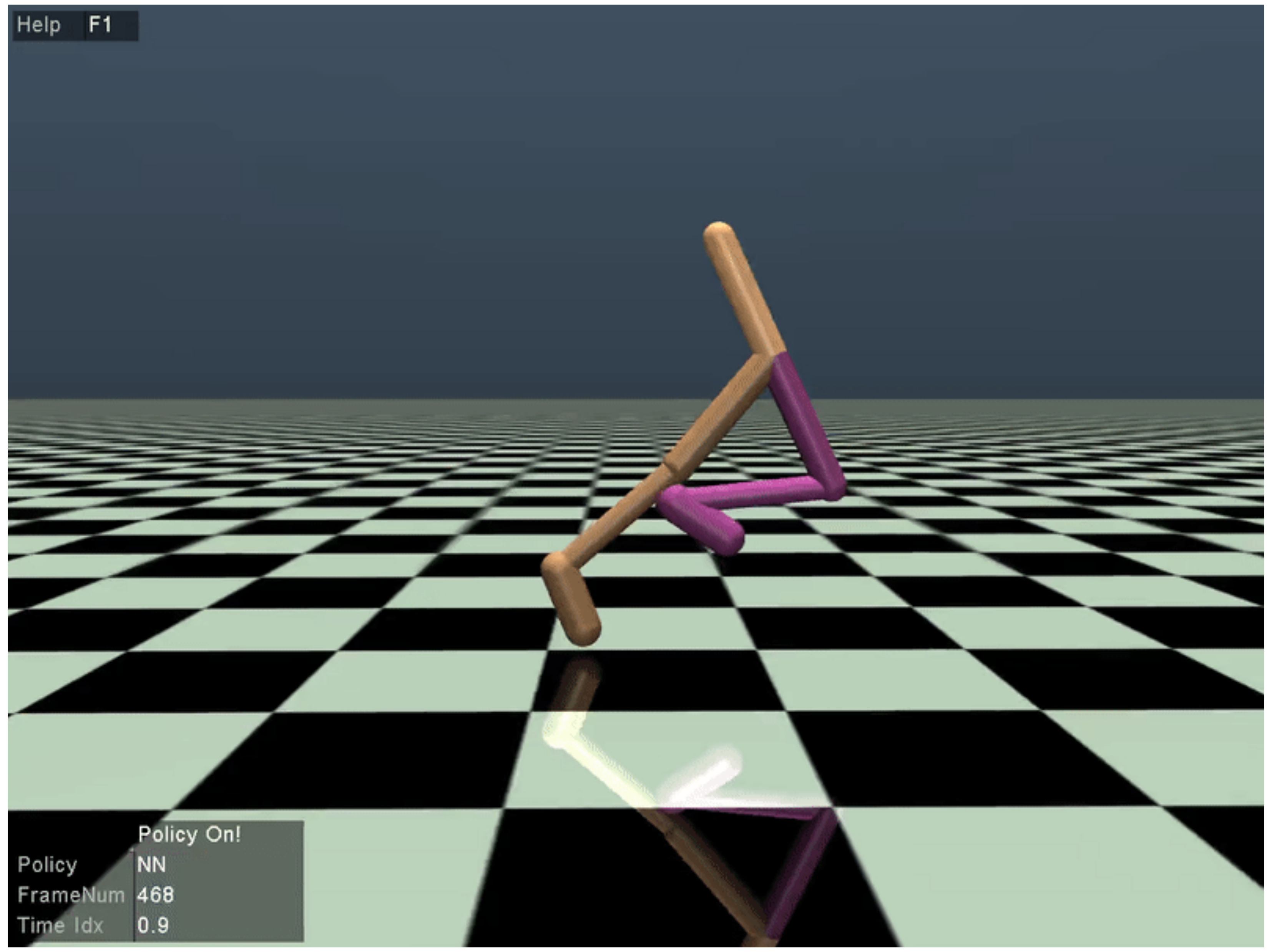

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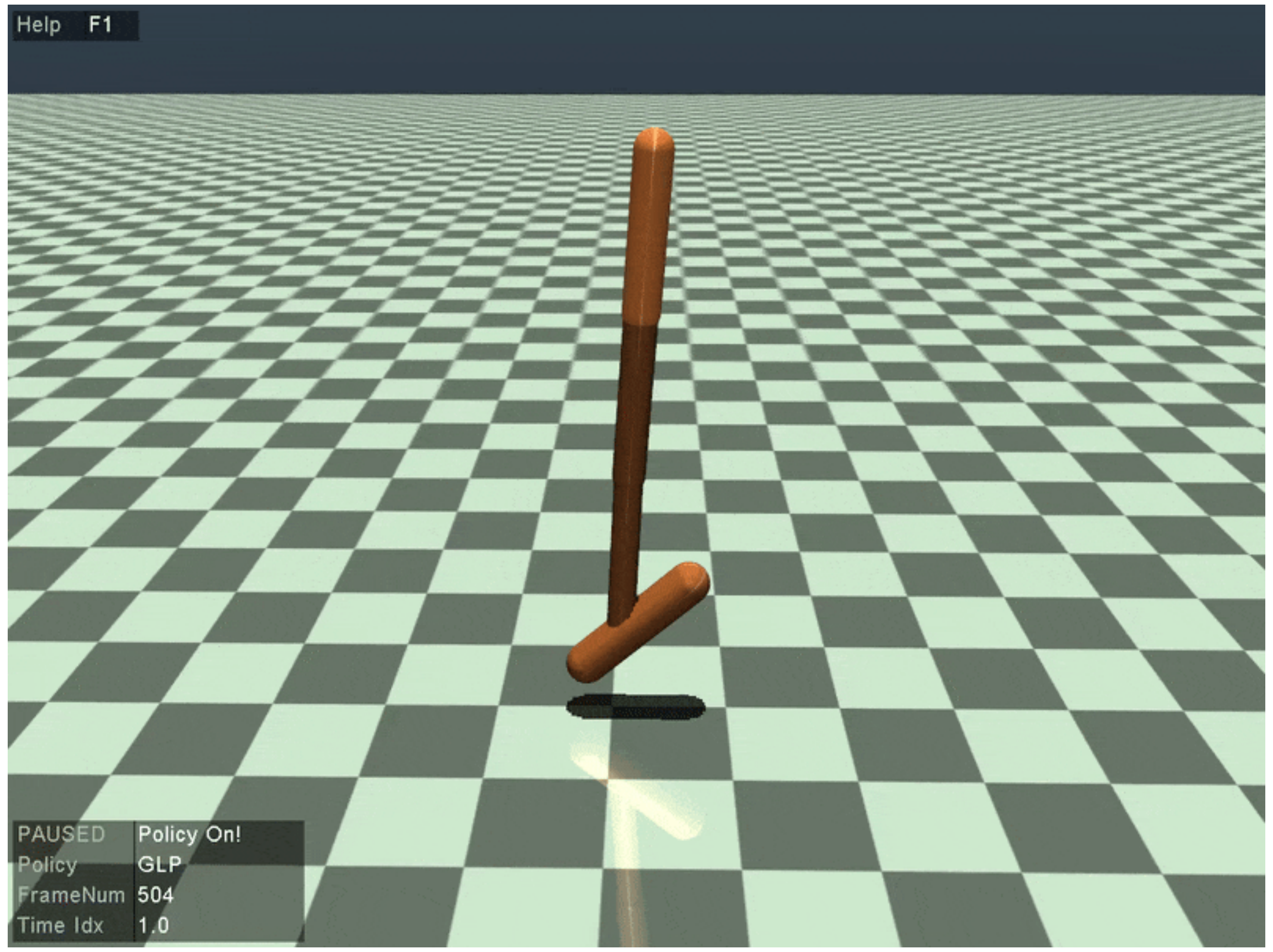

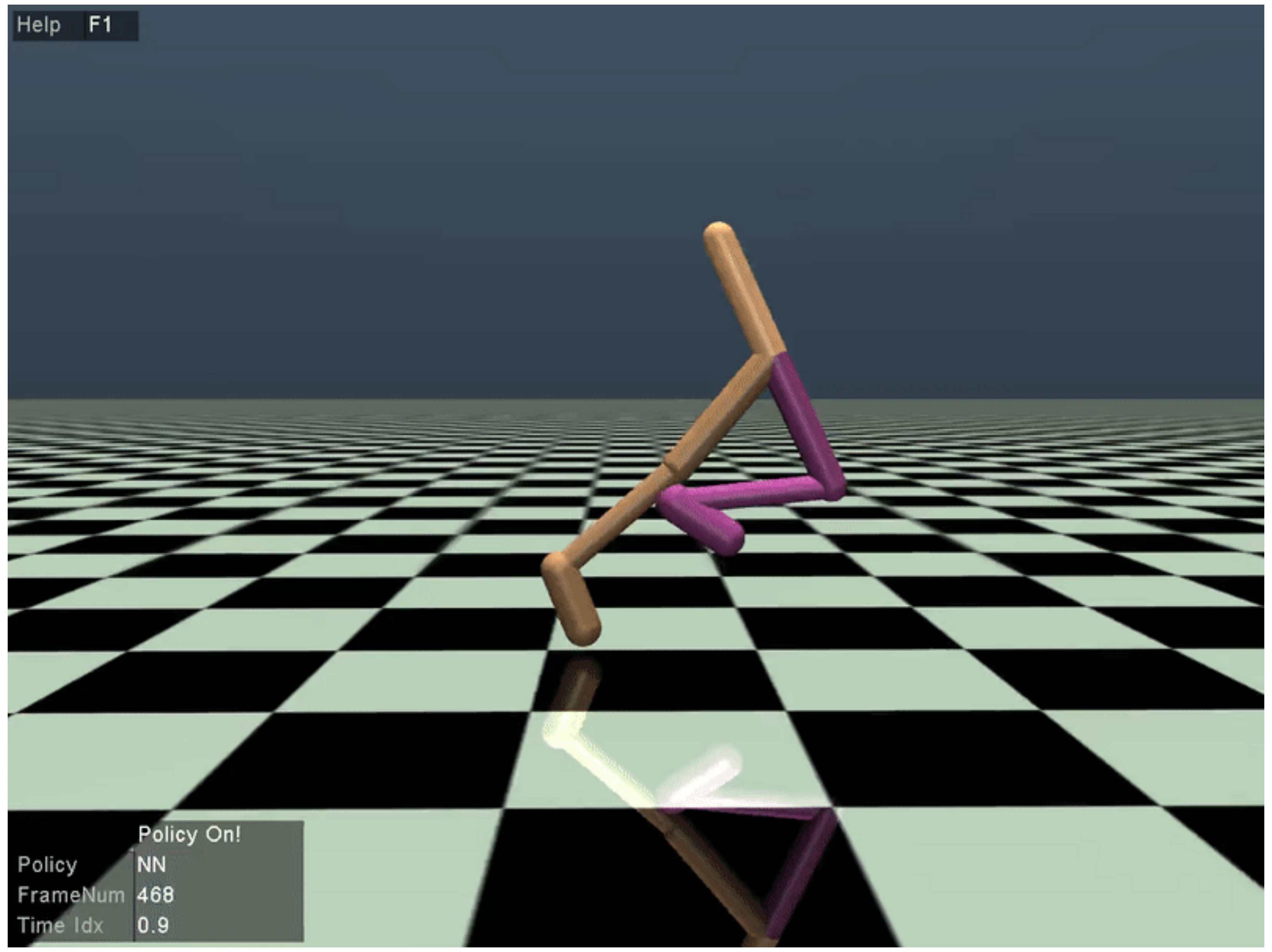

- [Rajeswaran, Lowrey, Todorov, K. 2017]: showed policies optimized for a single starting configuration s_0 are not robust!
- How to fix this?
  - Training from different starting configurations sampled from $s_0 \sim \mu$ fixes this:

$$\max_\theta \mathbb{E}_{s_0 \sim \mu}[V^\theta(s_0)]$$

Even if starting position concentrated at just one point—good for robustness!

OpenAI: progress on dexterous hand manipulation

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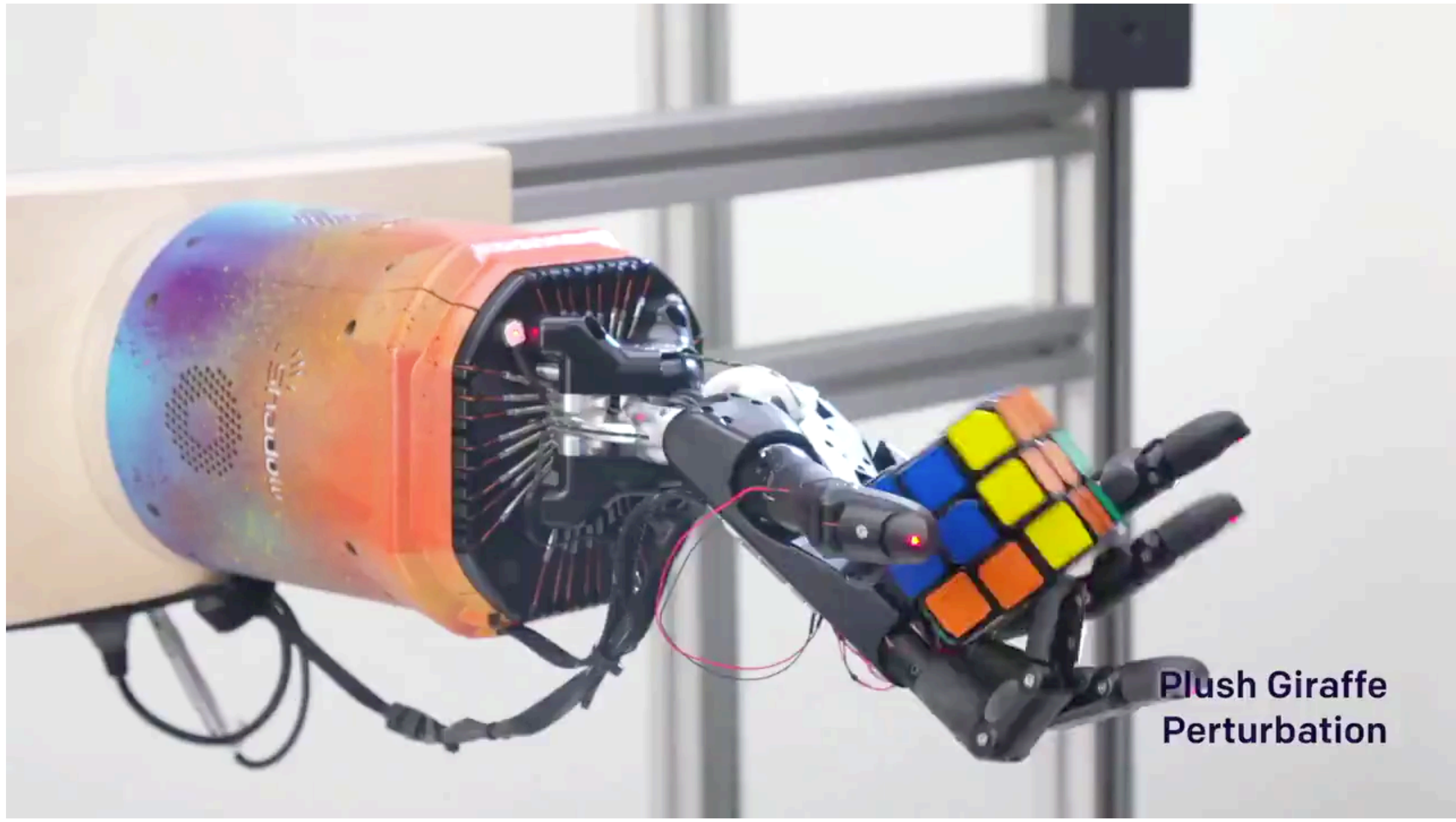
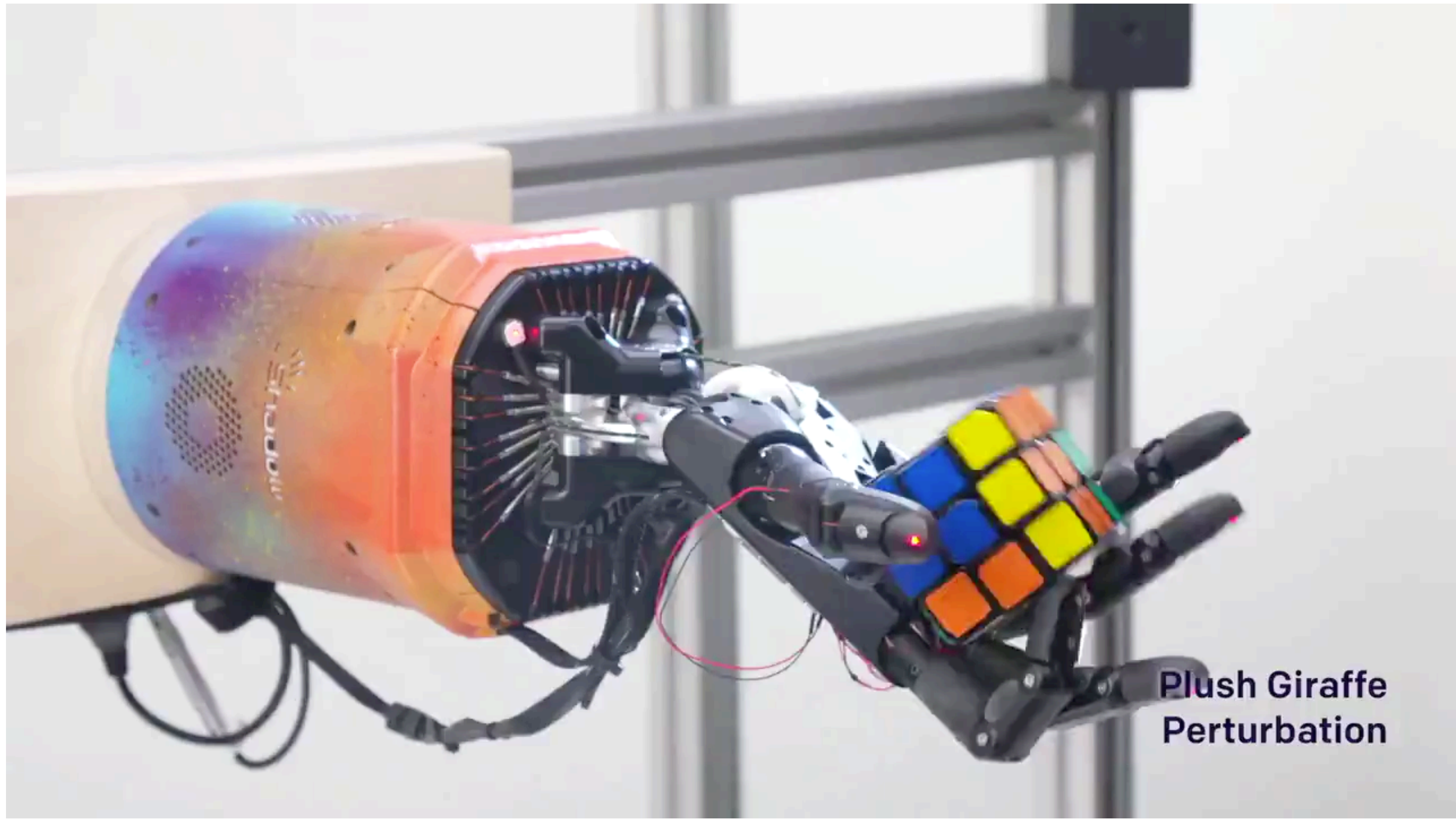


OpenAI: progress on dexterous hand manipulation



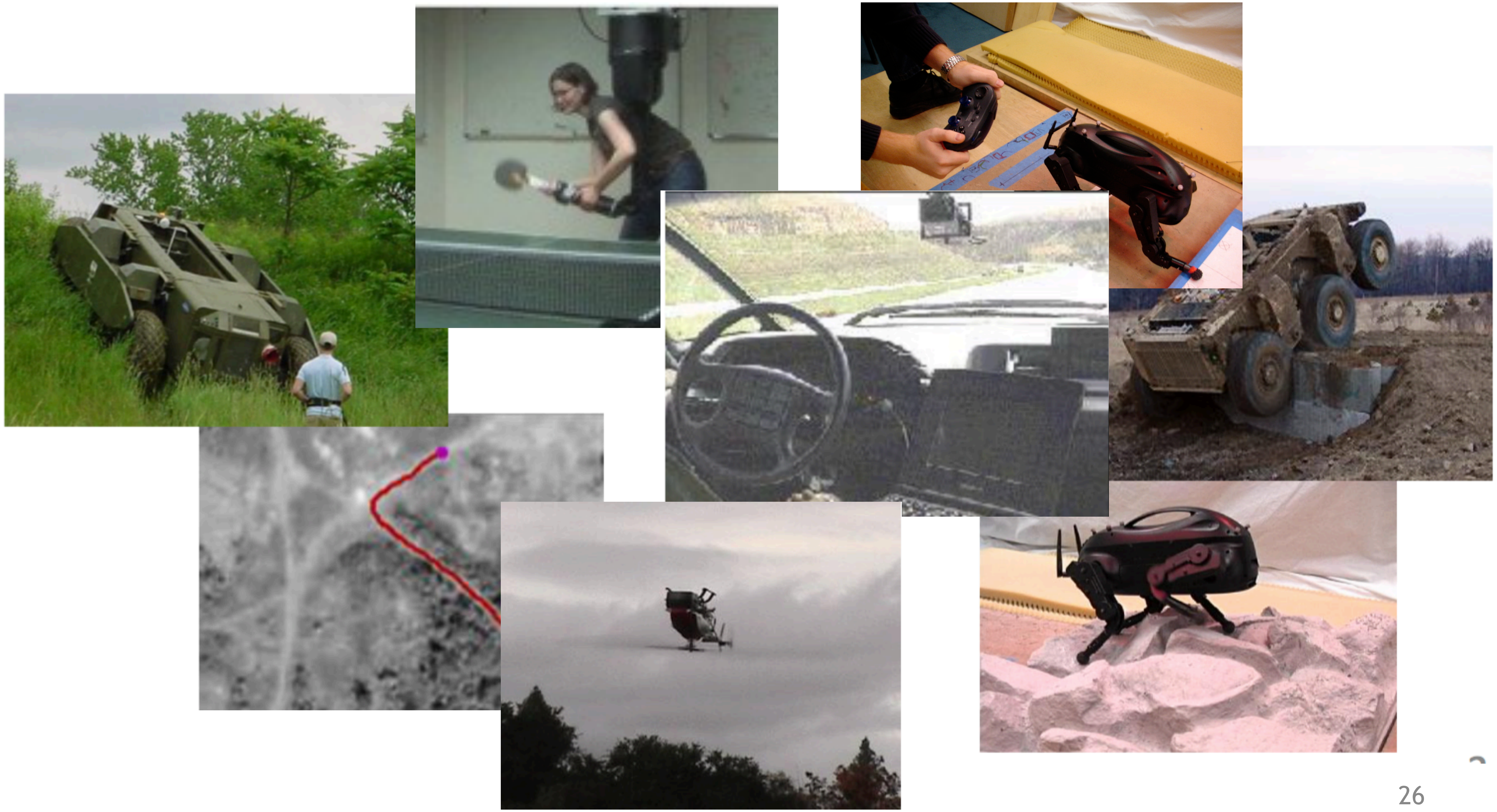
Trained with "domain randomization"

Basically, the measure $s_0 \sim \mu$ was diverse.

Today

- Feedback from last lecture
- Recap
- Importance Sampling (for PPO)
- PG review
- Exploration?

Imitation Learning



Imitation Learning

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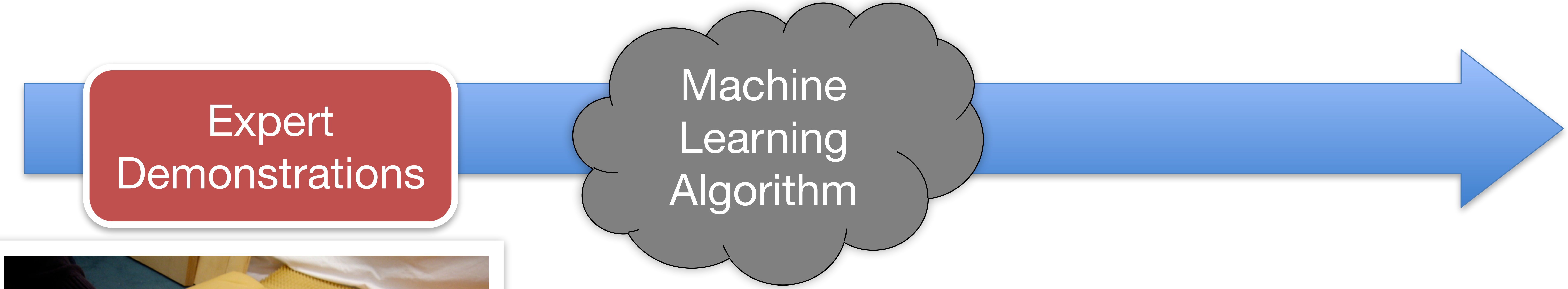
Expert Demonstrations

Imitation Learning



- SVM
- Gaussian Process
- Kernel Estimator
- Deep Networks
- Random Forests
- LWR
- …

Imitation Learning

Expert Demonstrations → Machine Learning Algorithm → Policy π

- SVM
- Gaussian Process
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- Deep Networks
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- LWR
- …

Maps *states* to <u>actions</u>

Learning to Drive by Imitation

[Pomerleau89, Saxena05, Ross11a]

Input:

Camera Image

Policy

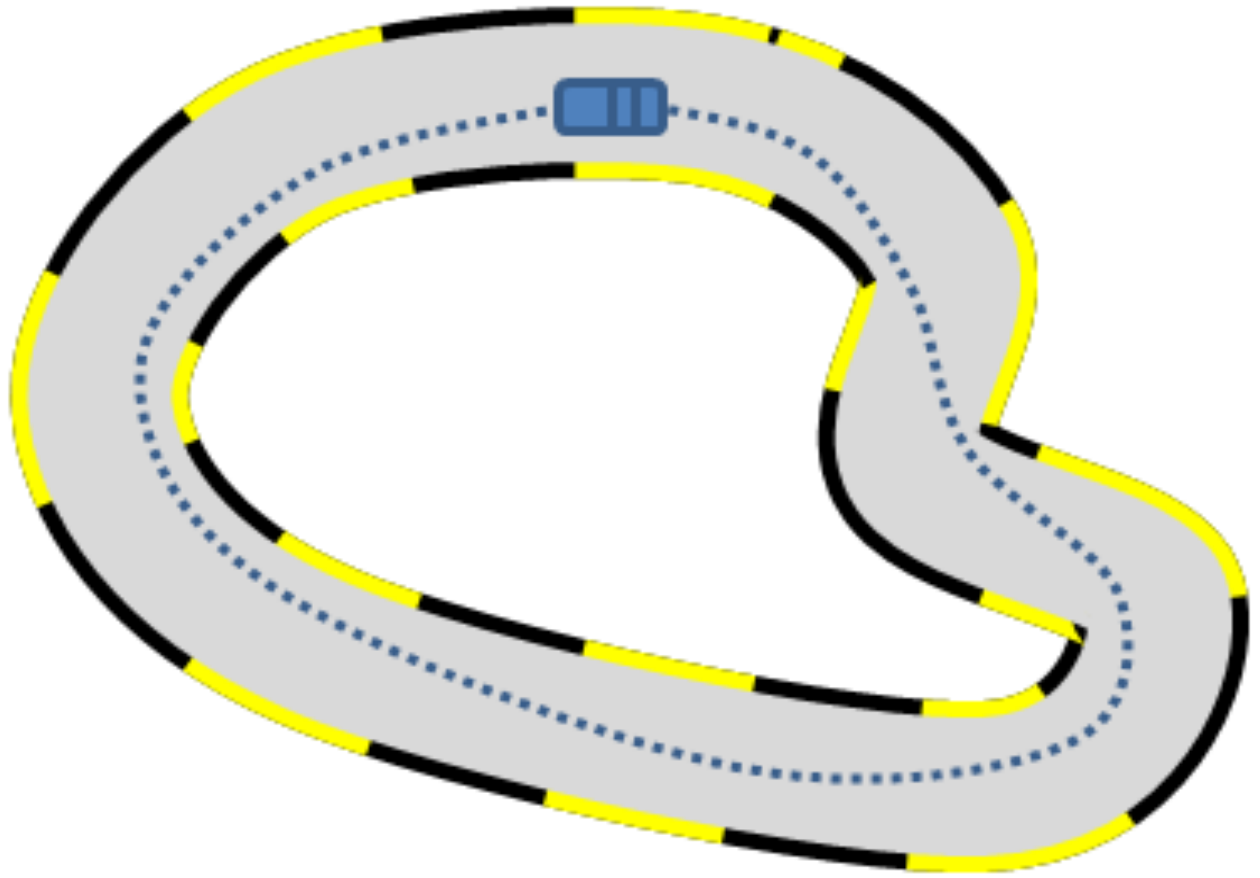
Output:

Steering Angle
in [-1, 1]

[Widrow64,Pomerleau89]

Supervised Learning Approach: Behavior Cloning

Expert Trajectories



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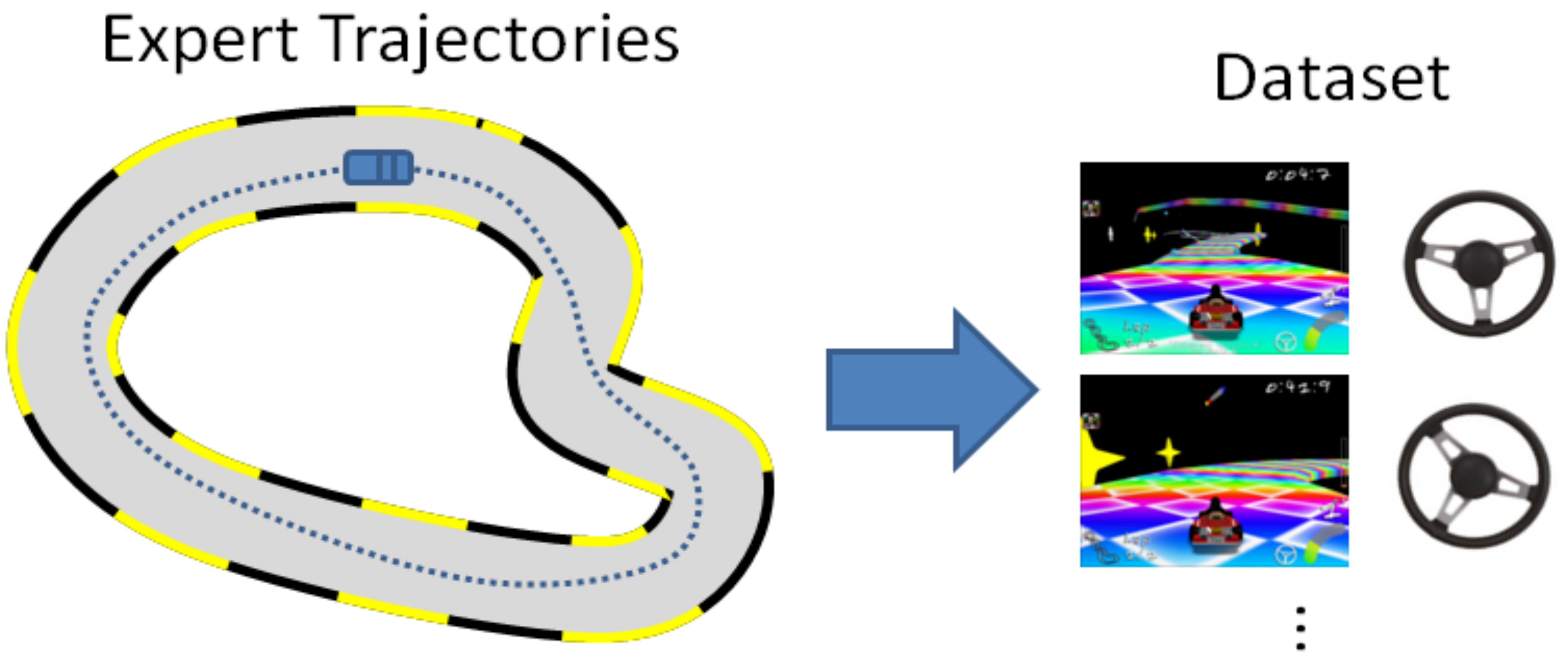

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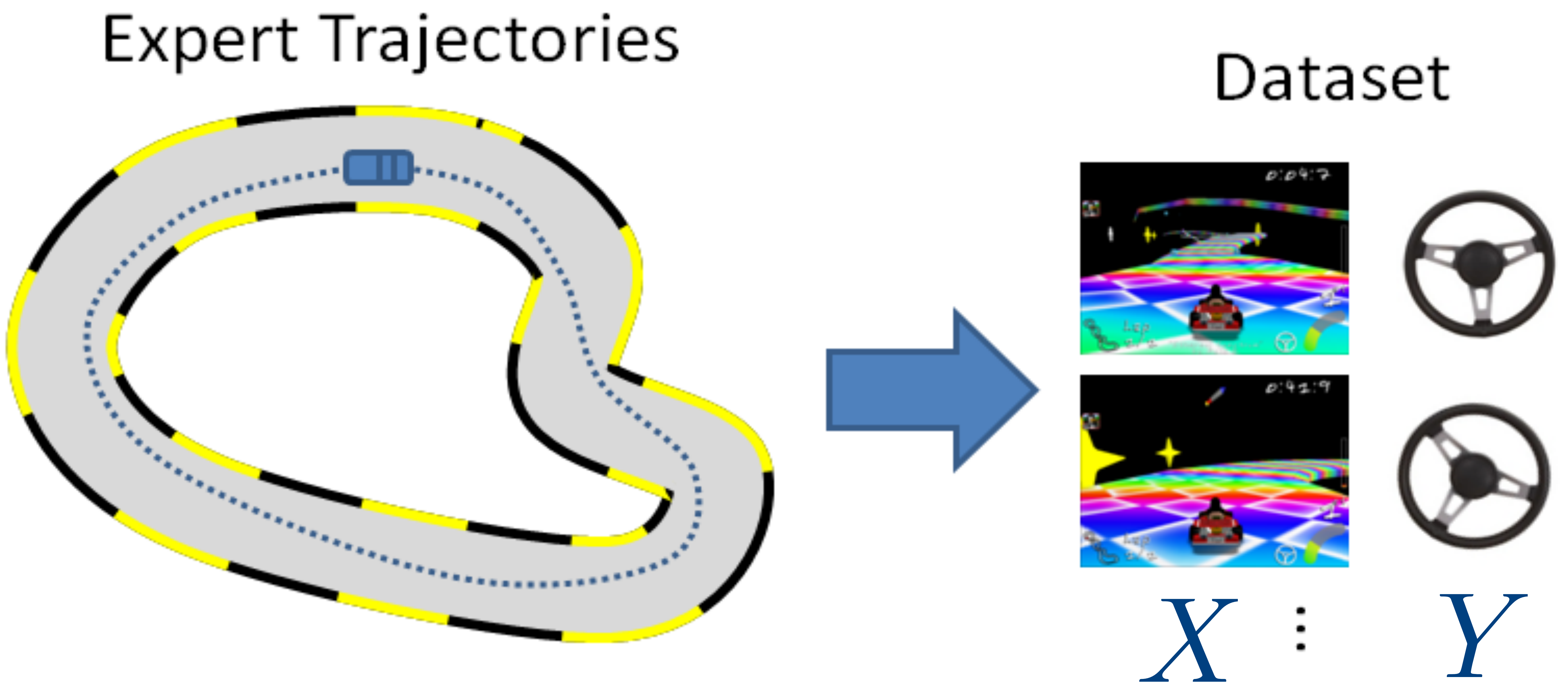

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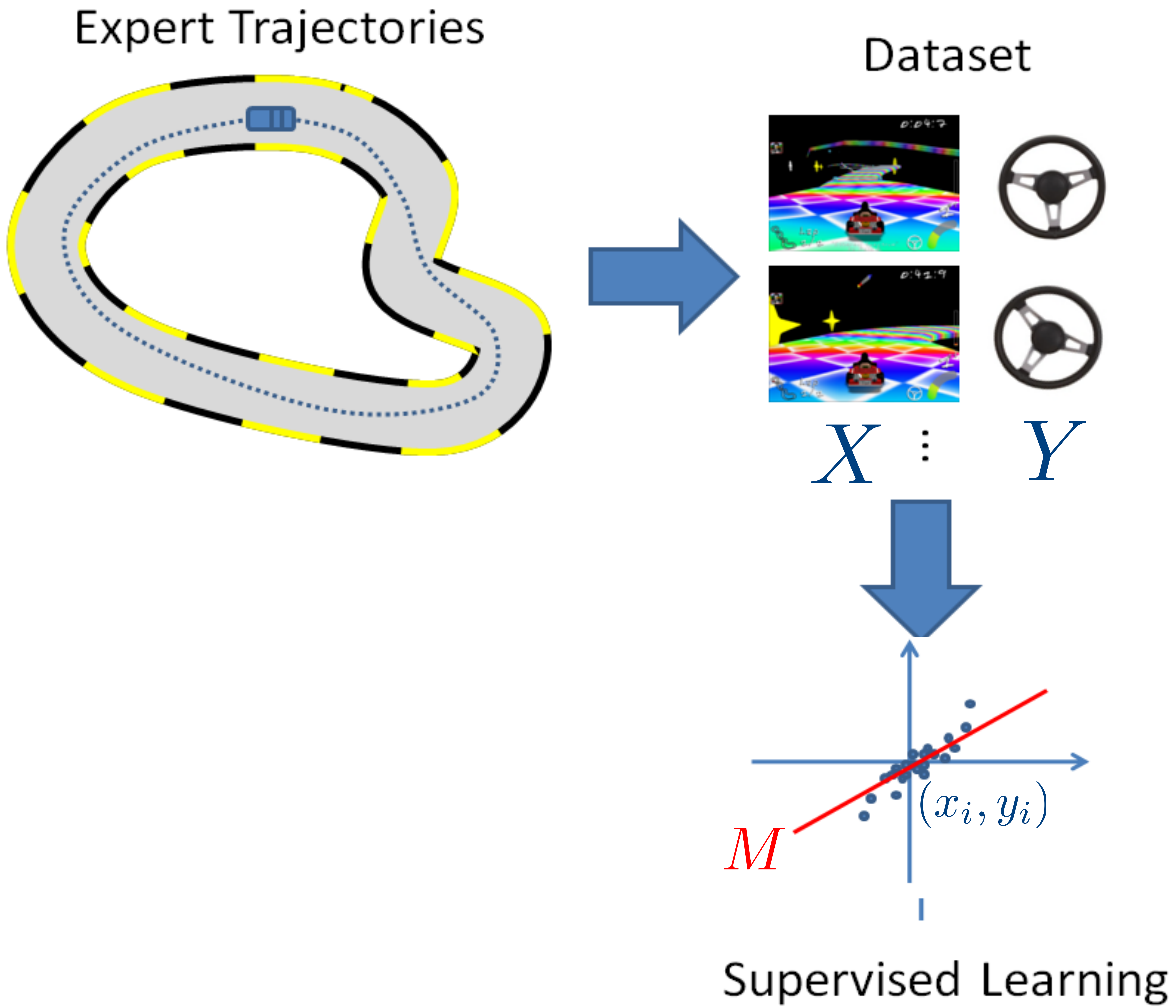

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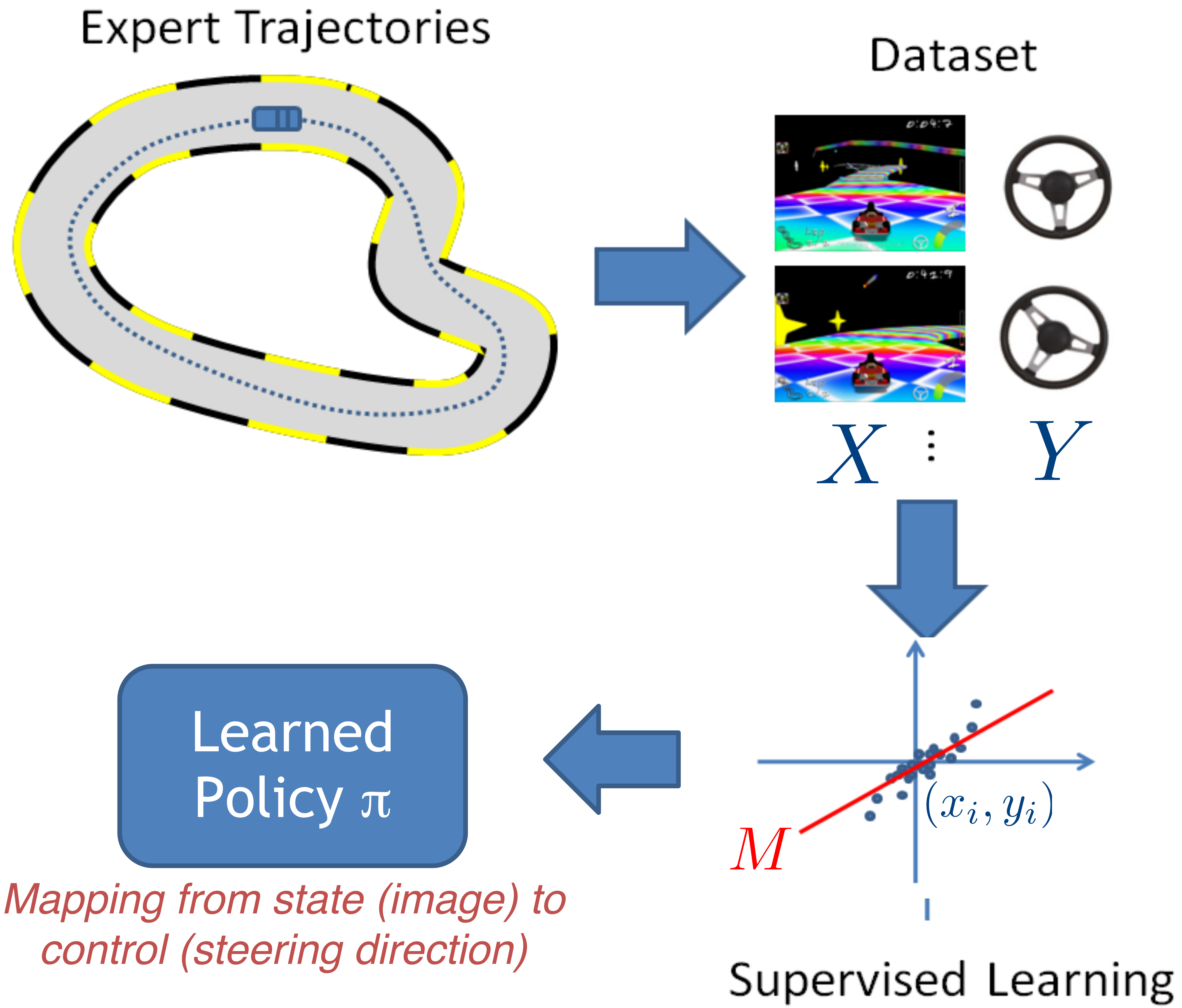
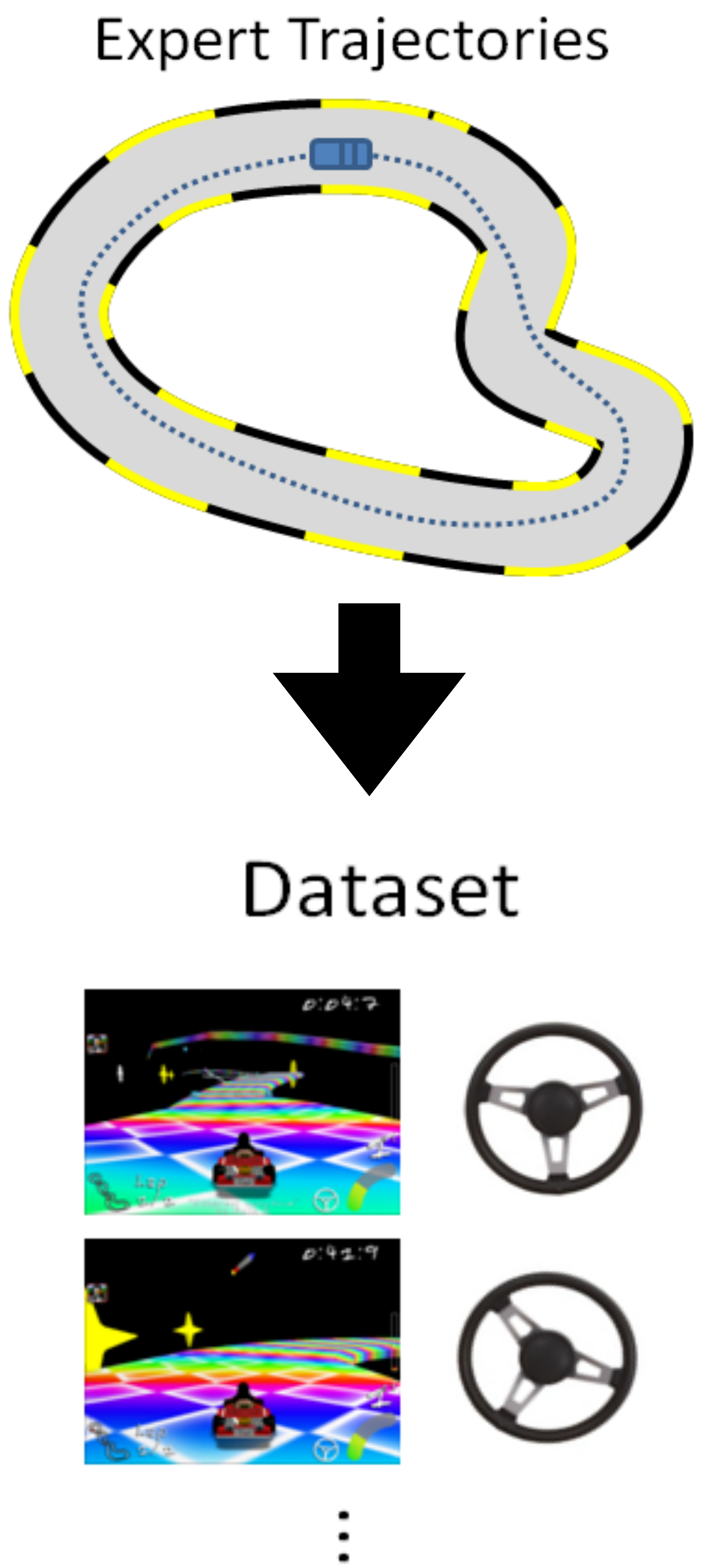


Mapping from state (image) to control (steering direction)

Let’s formalize the offline IL Setting and the Behavior Cloning algorithm



Finite horizon MDP $\mathscr{M}$

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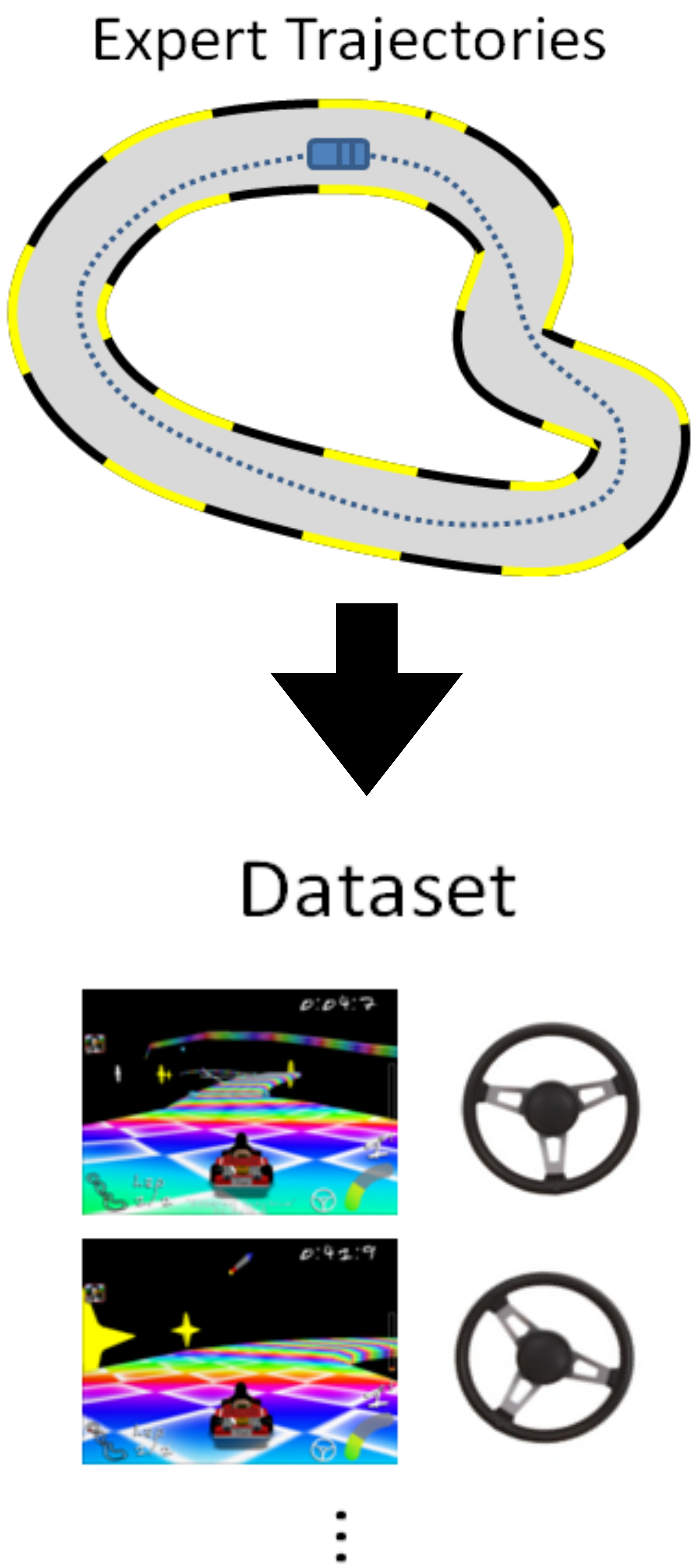

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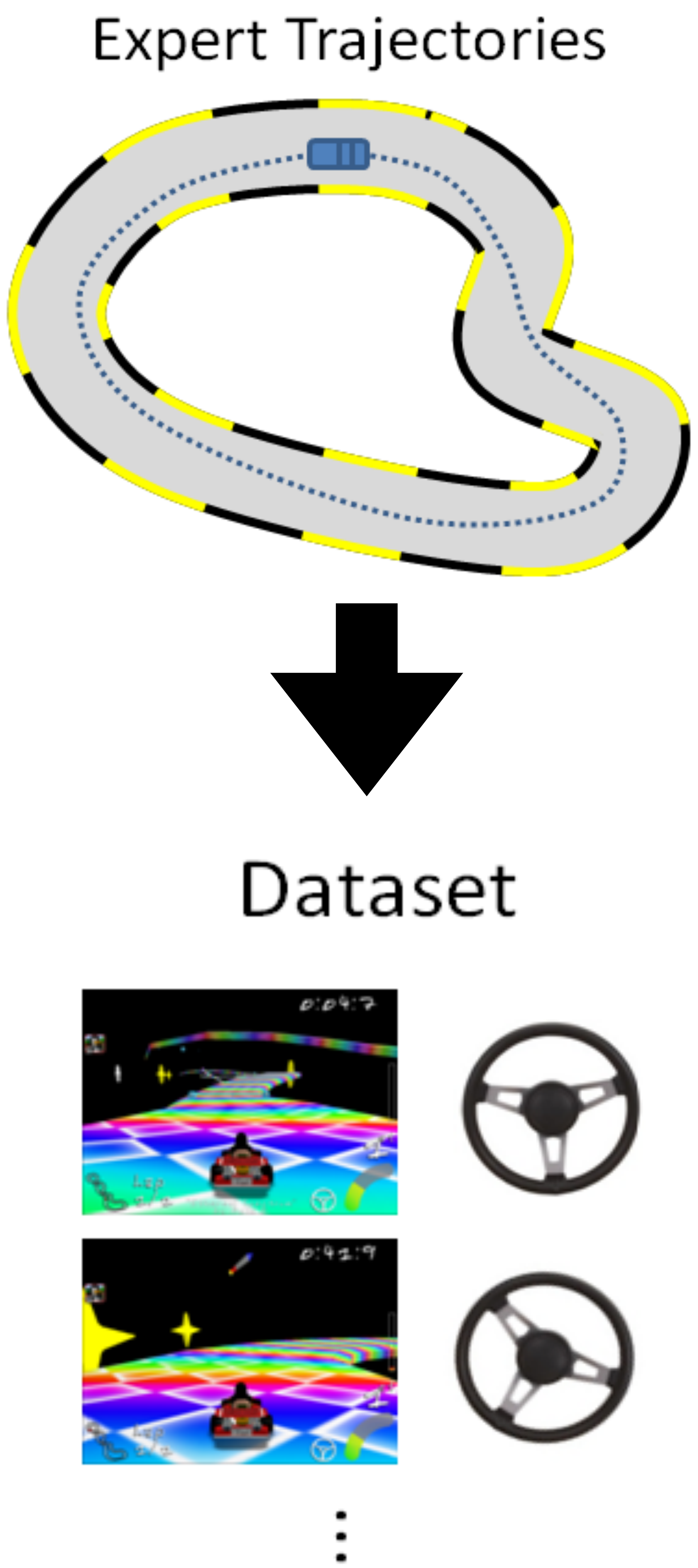

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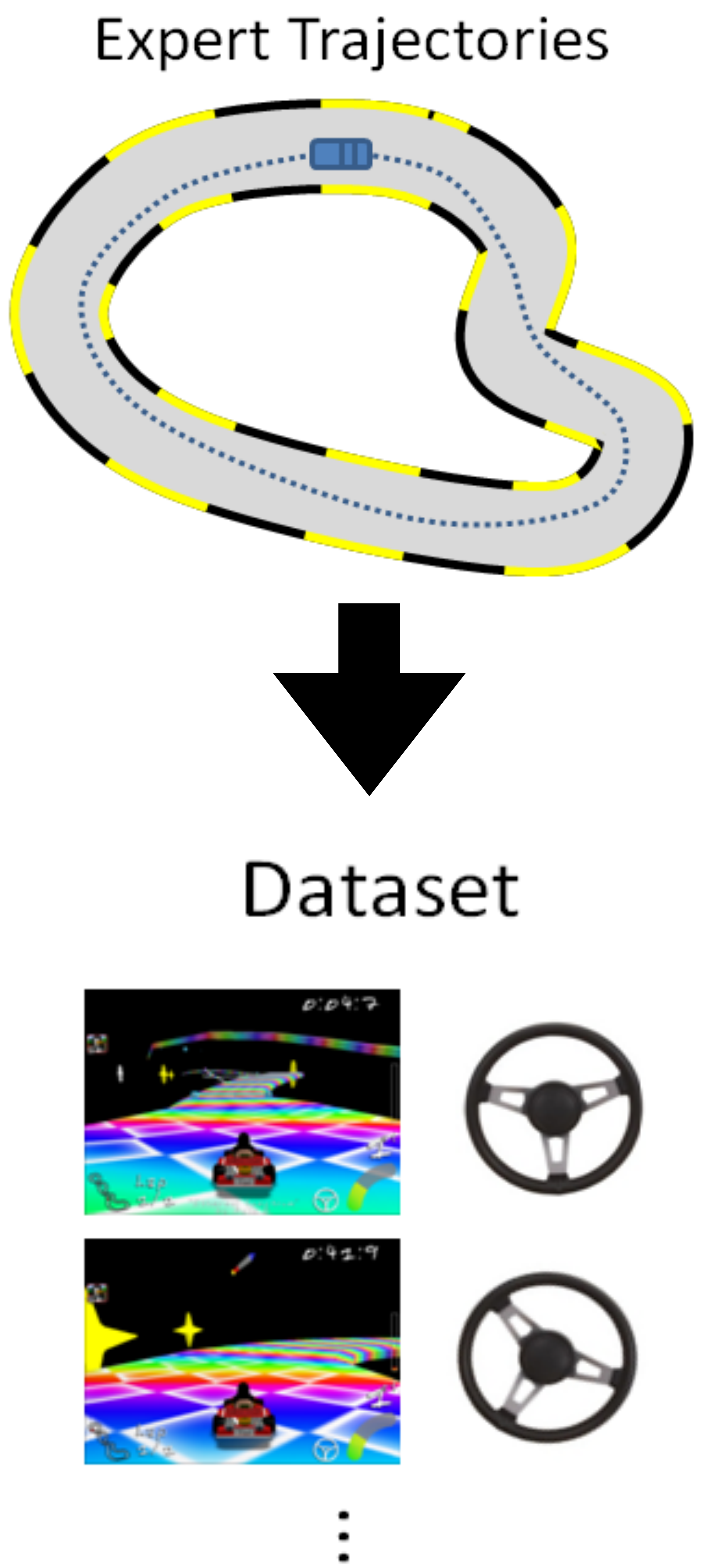

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Goal: learn a policy from $\mathscr{D}$ that is as good as the expert $\pi^\star$

Let's formalize the Behavior Cloning algorithm

BC Algorithm input: a restricted policy class $\Pi = \{\pi : S \mapsto \Delta(A)\}$

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3. square loss (i.e., regression for continuous action): $\ell(\pi, s, a) = \|\pi(s) - a\|_2^2$

Summary:

1. Importance sampling enables sample-based optimization in RL
2. Policy gradient methods are great and work well in practice, but can suffer from lack of exploration

Attendance:
bit.ly/3RcTC9T

Feedback:
bit.ly/3RHtlxy